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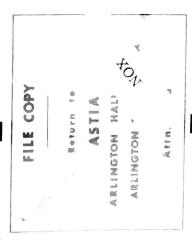
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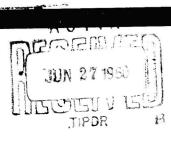
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### U. S. NAVAL ORDNANCE TEST STATION

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**NOTS TP 2298** 

WATER FLOW ABOUT A SWIMMING FISH

Ву

Moe William Rosen

Underwater Ordnance Department

This report is a thesis prepared for a Master of Science degree in Engineering at the University of California in Los Angeles. From the original printing of 190 copies, this document is

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May, 1959

U N C L A S S I F I E D

### FOREWORD

This report was originally prepared as a thesis for a Master of Science Degree in Engineering at the University of California at Los Angeles. The subject matter is pertinent to the work of the Naval Ordnance Test Station, and the author is a Station employee. In order to provide a larger distribution of the information than would be available from original printing reproduced at the author's expense, it is here re-issued in its entirety as a Station Technical Publication. It is hoped that the deviation from normal technical memoranda format will not detract from the technical content.

Wm. B. McLean Technical Director

### ACKNOWLEDGMENT

The author is indebted to Dr. Mark Cher of the University of California, Los Angeles, and Gerald Mosteller of the Naval Ordnance Test Station, Pasadena, California, for their invaluable assistance on several critical days of the experiments.

To my young daughter, Marcia Jill, goes my gratitude for her enthusiastic work with the many little fish and the raceway, sharing in the joy of initial discovery at home. And for proposing milk immersion, the method leading to success in the experiments, my thanks to my dear wife, Henrietta.

To Dr. Kenneth Norris of Marineland, California, go my thanks for his unfailing support of my efforts with the dolphins.

### **ABSTRACT**

A series of experiments were conducted with a very small fish in an attempt to determine the nature of the water flow about the swimming animal's variable form. The endeavors were failing until discovery was made of a very sensitive method which showed the entire field of water flow without inhibiting the action or the life processes of the fish. Another less sensitive method, the use of dye, also resulted in many failures but finally yielded two successful actions.

The experiments discovered a flow process and phenomena that are unfamiliar and surprising, involving discrete unit quanta of fluid energy. The observed flow is not in conformance with presently accepted ideas of laminar or of turbulent flow about a fish. The unfamiliar distribution and motion of the quanta seemed at first to be in contradiction to the fundamental laws of motion. The phenomena have resulted in a new Hypothesis of Fish Propulsion.

In one experiment, easily distinguishable particles flowed about the fish. Their motion was studied and found to be unusual.

Another experiment was performed in which the writer attempted the observation of a flow pattern about a dolphin as it leaped into the air, emerging at high speed from below the water surface. This experiment, requiring much patient effort, was successful and showed a remarkable flow similarity with the small fish experiments.

The observed behavior of the water on the small fish's body exhibits an astonishing correlation with the

actions of a fish when taken out of the water. These similarities tend to support the new hypothesis.

A method was developed, making possible the construction of the relative streamlines about the fish's undulating body. The lines are unorthodox.

In addition, a simple mathematical equation was derived relating the speed of a fish or dolphin to its body motions. The equation was reasonably verified by the measurements made in the experiments.

### CONTENTS

Foreword				•	iii
Abstract					v
Introduction					1
Gray's Paradox					2
Description of Experiments					7
Observations of Run I, A Strange Flow Made Visible					13
Birth of the Vortices					14
The Vortex Peg Hypothesis					25
Observations of Run II, With Sensitive Milk Layer .					26
Observations of Run III, With Dye					26 45
Secondary Vortices					46
Fluorescein-Dye Experiments	٠	•	•	•	46
Dolphin Experiment					48
Idealized Vortex Pattern	•	•			54
Continuous Charts of Motion, Runs I and II				•	56
Motion of Particles		•	•		56
How Are the Vertices Produced?					60
Structure of Vortex					62
Vortex Forces Acting on Fish					67 68
Static Force and Torque Balance					

The Streamlines About the Fish	69
Cylindrical and Hourglass Vortices	73
Evolution of a Vortex	73
Why Don't the Vortices Move?	77
Hypothesis of Propulsive Force From Scales of Fish	79
Calculating Drag From the Vortex	82
Equation of Fish Velocity	83
Test of Equation	86
Further Development of Equation	87
Conclusions	91
Pafaranaga	0.4

### INTRODUCTION

A little fish was gliding in the waters
Clear and deep
As he did swim I stared at him
At me he shyly peeped.
"Oh little fish," I asked of him
"How swimmeth thou so fair?"
He winked his eye and did draw nigh
And whispered in my ear,

"Thus and so, to and fro, oscillate I thither Undulate, integrate, from gills to tail I slither. Inundate, differentiate, twist molecules a-dither Multiply the waves do I, to take me you and hither."

Then having whispered thus to me,
He swung his tail quite gracefully
And swiftly did he swim to sea Leaping hither
Rippling whither
Dancing thither
Swimming, swamming, swum doth he.

M. Wm. Rosen 20 May 1959.

Perhaps this rhyme expresses close to the sum of our present knowledge as to how fish, and the cetaceans such as dolphins and porpoises, swim. This subject, the cause of much goodnatured amusement, may bring a smile to the reader. Let him be assured that it is both a fascinating and unprobed study. Just how does a fish swim? By wiggling his tail, of course.

In order to learn how fish actually do propel themselves, the writer began experimenting with live animals in July 1958. After a number of months of failures several experiments were suddenly successful. Surprising and unsuspected phenomena were discovered in the nature of the flow around a small swimming fish. These experiments are described herein.

### GRAY'S PARADOX

Why should we want to learn about the swimming of fish? The motive for these experiments stems from observations made by James Gray (Ref. 1). He pointed out what is, to this day, a paradox.

Gray computed the resistance of the water on a swimming dolphin, travelling at an observed speed. From this, the propelling power which the dolphin required was found. However, when this power was compared with that which the dolphin's muscles can develop, a wide discrepancy was found. It seems that biologically, the muscles of the animal are capable of only 1/5 to 1/10 of the power which well-established hydrodynamic laws demand.

To illustrate this, the writer has made his own computation on the dolphin, based upon the original measurement of speed which Gray used. The following is Gray's Paradox.

In the Indian Ocean E. F. Thompson timed a dolphin with a stop watch while it was passing his 136-foot ship from stern to bow in 7.0 seconds. The ship was sailing at  $8\frac{1}{2}$ -knots logged speed, and the dolphin's path was about 30 feet from the ship's side. Algebraic computation establishes the speed of the dolphin as 33.8 ft/sec or 20.0 knots. Faster speeds are claimed for this remarkable animal, but let us use this number for the present calculation.

L = 6 feet, estimated length

S = 15 ft<sup>2</sup>, wetted surface of a 6-foot dolphin, measured from a model

V = 33.8 ft/sec speed (observed)

W = 200 pounds, weight of a 6-foot dolphin

m = 35 pounds, muscle weight of a 6-foot dolphin (biological measurement)

The laws of fluid dynamics state that the drag, or resistance of a fluid to the motion of a streamlined body, is given by the equation

$$D Drag = C_d \cdot w \cdot \frac{V^2}{2g} \cdot S$$

where

Cd = coefficient of drag based on wetted surface

w = density of fluid (64.0 lb/ft<sup>3</sup> for sea water)

g = acceleration of gravity, 32.174 ft/sec<sup>2</sup>

The coefficient of drag Cd has been measured by a great many experimental data, which are shown in Fig. 1. This chart gives Cd when the Reynolds number R is known.

where  $\sqrt{V} = 1.31 \times 10^{-5}$  ft<sup>2</sup>/sec for sea water at 60°F.

Note that in some regions of Fig. 1 there can be laminar flow or turbulent flow. Laminar flow is a smooth, parallel, streamlined type of fluid motion. It has a considerably lower drag coefficient than turbulent flow in which the fluid is disturbed,

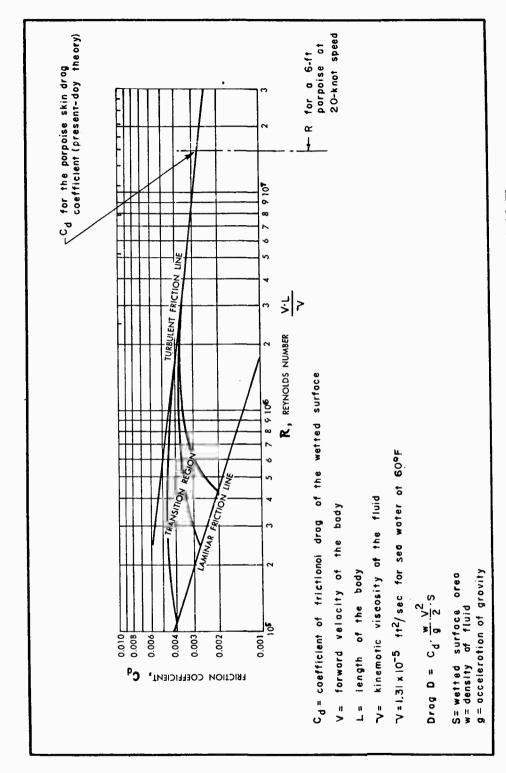


FIG. 1. Surface Drag Coefficients per Present-Day Fluid Theory.

4

turbulent, and mixed by irregular motions and eddies, in addition to the general flow direction.

For the dolphin above

$$R = \frac{33.8 \times 6.0}{1.31 \times 10^{-5}} = 1.55 \times 10^{7}$$

From this, Fig. 1 gives the skin frictional drag coefficient as

$$C_{d} = 0.0028$$

There should be added a small increase due to "form" drag of the dolphin's three-dimensional but well-streamlined body. Add 0.0002.

Then

$$C_{d}$$
 (total) = 0.0030

The total drag may now be computed.

$$D = C_{dw} \frac{V^{2}}{2g} \cdot S = \frac{0.0030(64.0)(33.8)^{2}15}{2(32.174)}$$

= 51.1 pounds water resistance

The drag power is the product of drag and velocity.

Drag power = 
$$\frac{DV}{550} = \frac{(51.1)(33.8) \text{ ft lbs/sec}}{550 \text{ ft lbs/sec/hp}} = 3.14 \text{ hp}$$

Assuming that the dolphin's ability to convert muscle power to drag power is at least as good as a ship's propeller:

e (propulsive efficiency) = 85%

Then

$$Muscle power = \frac{3.14}{e} = 3.7 \text{ hp required}$$

and

Power required per lb of muscle =  $\frac{3.7 \text{ hp}}{35 \text{ lb}}$  = 0.106 hp/lb muscle required, at 20-knot speed

It is this figure of 0.10 hp/lb muscle that creates the paradox which Gray recognized. According to measurements (Ref. 2) made of the power of mammalian muscles (dog, man) it seems that the tissue is capable of only 0.01 hp/lb muscle at a steady sustained rate. Man can develop a temporary 0.024 hp/lb muscle (Ref. 3) but only for a limited time before exhaustion. How then can the dolphin swim steadily at 20 knots if it has only one quarter to one tenth of the required power to do so? Or conversely, if it does have sufficient power for this speed, then it is four to ten times greater than man's per pound of muscle.

Now if the dolphin can swim faster than 20 knots (and many experienced seamen say it does) then the discrepancy becomes even greater. For instance, if it can swim at 25 knots (a guess only) it will require:

$$\left(\frac{25}{20}\right)^3$$
. (0.106) = 0.207 hp/lb muscle

since the power required for propulsion rises as the cube of the speed. This would mean development  $8\frac{1}{2}$  to 21 times the power which the physiological measurements in man indicate as possible; and the ability of the dolphin's respiratory system to absorb and use oxygen would have to be far beyond that of known biological capacity in man.

An explanation is offered by some in this way. If the porpoise can maintain smooth laminar flow over its entire body, then the coefficient of skin drag at 20 knots diminishes to about 0.0003. Therefore (if it also had almost no "form" drag), the power required to propel itself through the water would be much smaller, by the ratio

$$\frac{C_{d} \text{ (laminar)}}{C_{d} \text{ (turbulent)}} = \frac{0.0003}{0.0030} = \frac{1}{10}$$

This, it is pointed out, would take care of the paradox.

Figure 2 shows this conception of smooth, non-turbulent laminar streamlines over the entire body of a fish, enabling it to slip through the water with almost no disturbance. Unfortunately, if this is true, then we have substituted a new paradox for the first one. At a Reynolds number of  $1.55 \times 10^7$  nature has not yet permitted us to observe a fluid flowing in a completely laminar fashion.

Similar calculations made on large fast fish moving at claimed or observed speeds yield the same puzzling conflict between the biological and fluid dynamic sciences. That certain fish can move very rapidly is gradually being verified. D. R. Gero (Ref. 4), for instance, measured accurately the speed of a barracuda at 40 ft/sec (27.3 mph).

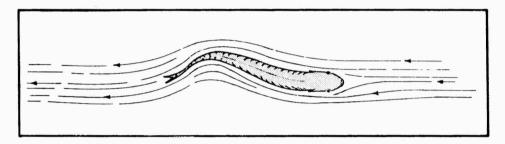


FIG. 2. Previous Concept of Smooth Laminar Flow (Non-Turbulent Vortex Free Flow Over Body of Fish).

### DESCRIPTION OF EXPERIMENTS

The purpose of the experiments was to make visible the lines of flow about a live normally swimming fish by the use of dye or particles. Many beautiful photographs have hitherto been made of fish, but none showed the action of the transparent water. If this flow could be clearly and precisely observed, it was reasoned that perhaps the answer to the paradox could be found. Flow lines have been made visible in wind and water tunnels about rigid, non-living objects such as aerodynamic wing shapes, fuselages, and the like. However, attempting the same thing with a living jumping wriggling fish, whose shape while swimming is changing at every instant, proved to be quite a different thing.

It was decided that small tropical fish might make good subjects. Accordingly in June 1958 some first attempts were made. In the struggle of man against fish, the fish won out. Dyes of many kinds, including food dyes, either weakened the fish so that they would not swim normally, or killed them. In addition, no dye would stay on a swimming fish in sufficient concentration for more than a fraction of a second. It was learned after a time that a long channel was the best vessel to direct the fish's motion. Also it was found that only a few species of tropical fish had body and fin forms similar to the large swift fish of interest. The fastest tropical species was "Pearl Danio," Brachydanio albolineatus. Figure 3a shows the individual which seemed at the same time the swiftest and most cooperative, little Geronimo, 15/8 inches long.

The vessel, as it evolved, is a metal channel raceway  $41\ 3/4$  inches long,  $1\frac{1}{2}$  inches wide, and I inch deep. A portion of the bottom is a sealed transparent plate with accurate grid markings. The fish is held imprisoned in one end by a gate. At the proper moment the gate is lifted and the fish is supposed to race down the channel, into the camera's field of view. Most often it swims erratically rather than steadily, or does not start at all, or turns back after a few inches of travel.

Little success rewarded the first months of effort. Then an unexpected accident provided the solution. As one of the "dyes," milk was being tried by immersing the fish in it for a moment and then, when the fish swam into clear water, attempting to observe the milk as it streamed off. The milk did not stay on for more than a fraction of a second, and the streamers were too weak to be clearly seen. After one of the trials the fish sulked in one end of the race for several minutes. It then decided to swim back to the other end which was clouded with milk. A clear pattern of some kind flashed to the eye for an instant and then became indistinguishable.

The milk had partially settled into a thin bottom layer. As the fish swam over it, water currents traced their paths on this layer, exposing the black race beneath. We later found that when the milk was injected very gently at the bottom with a glass tube, it would settle in a uniform layer which was extremely sensitive to the weakest water motions. Even a gentle touch of the surface

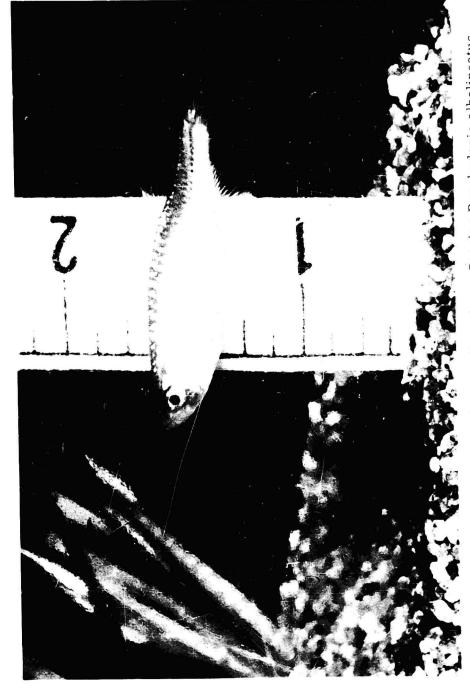


FIG. 3a. The Hero of the Experiments, Geronimo. Species Brachydanio albolineatus.

with the tip of the finger one inch above this layer produced a momentary dark spot on the bottom. With this discovery we progressed more rapidly, since milk did not seem toxic to the fish. Each trial required flushing out the race, filling it with clear water of proper temperature, waiting for quieting, and carefully injecting the milk. The setup is shown in Fig. 3b.

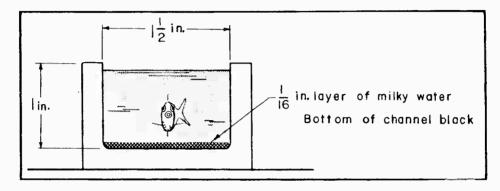


FIG. 3b. Cross Section of Channel Race in Experiments.

The first successful pictures were taken with an 8-mm movie camera, with the able assistance of the writer's young daughter Marcia. The pictures convinced us that we were indeed seeing something unexpected and extraordinary. Accordingly it was decided to construct the apparatus needed for high-speed motion-picture photography. In Fig. 3c and 3d this apparatus is shown. The camera views the channel by reflection from a large special mirror at 45°, and takes 35-mm motion pictures at the rate of 100 per second. The light is provided by two Edgerton lamps synchronized with the camera so as to flash 100 times per second, each flash occurring at the precise moment that the film is stopped by the mechanism. The intense flash of the lamps lasts two millionths of a second. The experiment is conducted in darkness, broken by the blue-white burst of light during the several seconds the camera, lamps, and fish are operating.

In addition to the method of the sensitive quiescent milk layer, a successful dye experiment was also made. The race bottom has a transparent window with 0.25-inch grid markings. Edgerton light is diffused and reflected up through this grid into the mirror (Fig. 3d, left) and thence into the camera lens. The

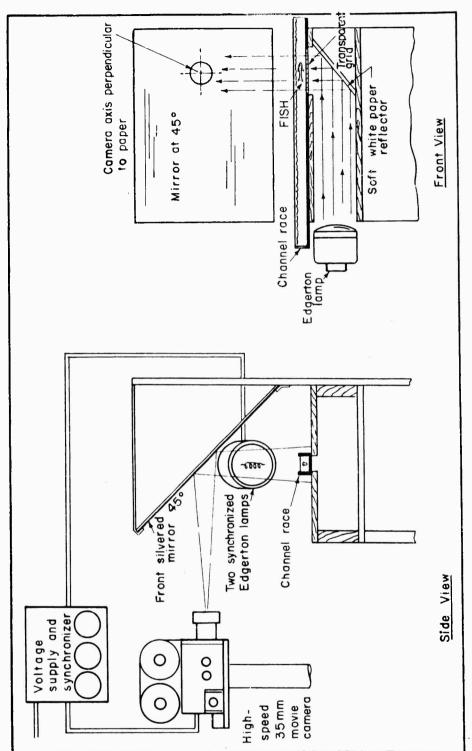


FIG. 3c. Setup of Apparatus for Experiments.

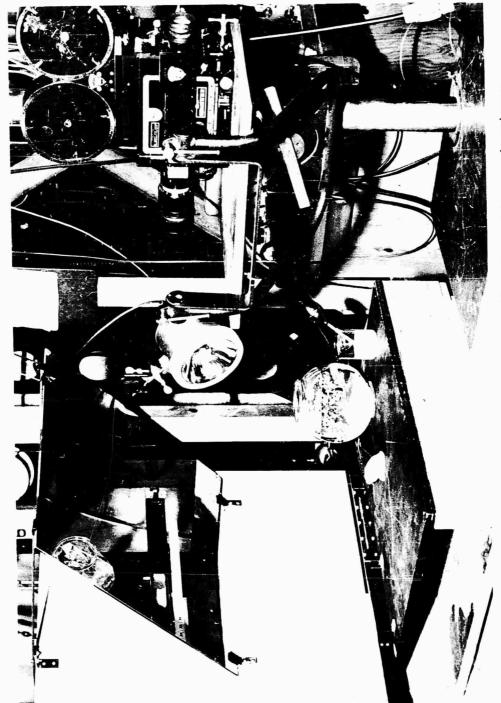


FIG. 3d. View of Apparatus. Transparent grid in channel is seen in mirror.

fish is immersed in a solution of methylene blue dye and transferred to the channel. A large number of patient attempts were finally rewarded when the fish shot across the transparent grid under its own power within one second of dye dunking, leaving a visible trail, while the camera and lamps were running.

In all, about 17 trials were made with the 8-mm home camera, and 34 trials with the 35-mm special apparatus. Of these, three trials were successful. These are identified as Runs I and II with quiescent milk, and Run III with methylene blue. The star performer of all these runs was little Geronimo, the swiftest and toughest in the bowl.

### OBSERVATIONS OF RUN I, A STRANGE FLOW MADE VISIBLE

In this trial the little fish reached a speed of 24.0 in/sec over the sensitive milk layer. When the film is projected as a movie, a beautiful pattern and trail are seen in motion, as well as the undulations of the fish's body. When enlargements were made of the individual frames, the full meaning of what was seen did not become clear until many hours of examination. These pictures, presented in Fig. 4 through 8\* show some remarkable phenomena taking place.

The milk responds to the complete field of disturbance to some distance on either side of the fish, beyond the boundary-layer fluid.

The trail created by the fish is a beautiful system of large, slow, spiralling vortices. At first it was thought that this was a "Karman vortex street," a series of vortices produced by blunt bodies moving through fluids. However, this was not correct. A careful graph made of the main milk vortex centers shows only a single row, with the direction of rotation reversing from one

<sup>\*</sup> Due to the limitations of photo processes, the reproductions of these plates cannot show all the fine detail present in the original.

vortex to the next. This row follows the path of the fish's head. Thus if the fish had followed a perfectly straight average path, these vortices would lie on one straight line. A Karman vortex street has two rows separated by a distance 28% of the vortex spacing.

A remarkable property of the milk vortices is that once the fish has passed, their centers do not move appreciably in the direction of swimming or in the reverse direction. In addition, no net over-all water current moving in the rearward direction can be discerned when the fish is swimming steadily, a fact which is hard to accept, yet exists. At one point, when the fish is accelerating in Frames 23 to 26 (Fig. 6), an arm of Vortex H expands obliquely to the rear. This is the only observable occurrence of an appreciable rearward current and must be associated with acceleration. The vortex arms, as they lengthen out, should not be confused with the centers. The non-movement of the vortex centers differs from the Karman vortex street in which the vortices follow the body at a speed of about 1/6 that of the body.

Some of the swirls bear a marked resemblance to the beautiful astronomical spiral galaxies or "island universes" found in space--each consisting of billions of stars rotating in a gigantic spiral wheel, millions of light-years in diameter. Many of the tiny swirls in the milk layer possess, like the galactic nebulae, two spiral rotating arms which wind out from the center. Good examples of these are Vortices F and G in Frame 26 (Fig. 6). Another beautiful example is Vortex K in Frames 39 and 40 (Fig. 8).

Small secondary curls or vortices are sometimes formed at the ends of the arms of the main vortex, just as in the astronomical galaxies. These secondary curls form a zigzag 60° pattern which is hard to see in this run.

### BIRTH OF THE VORTICES

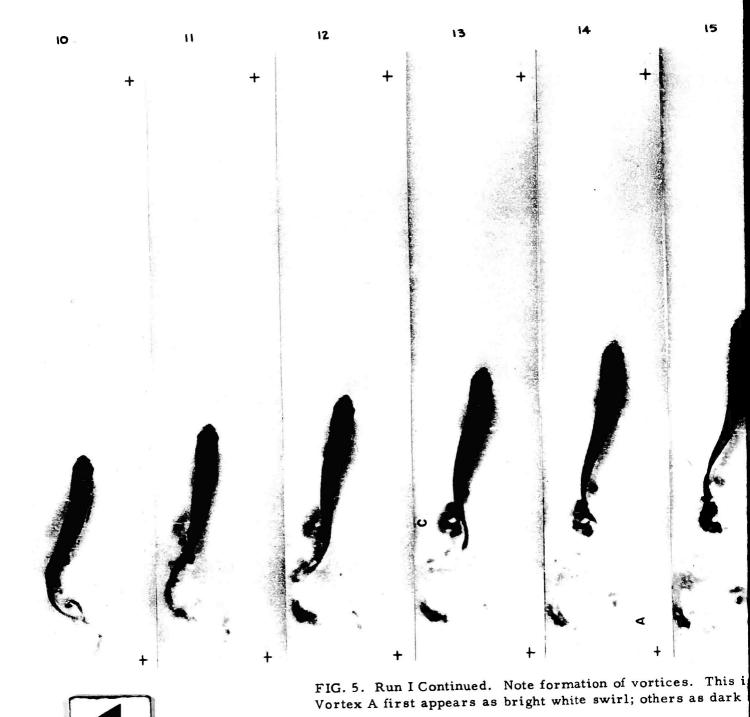
The pictures of Run I also show another striking phenomenon: the birth of the individual vortices as they form out of seemingly nothing, first on one side of the fish--just aft of its gills, and then on the other. As the fish swims on, each vortex grows in

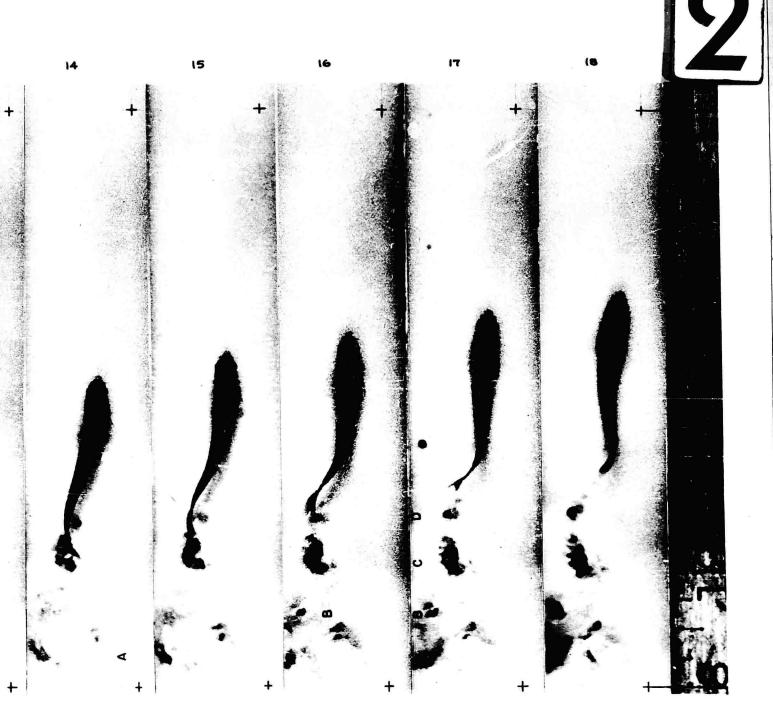
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FIG. 4. Run I, Fish Swimming at Speed of 24.0 to 18.8 in/sec. Exposure time 2/1,000,000 sec. Marks (+) establish identical points.



t Speed of 24.0 to 18.8 in/sec. Time between frames 1/100 sec. Marks (+) establish identical points on all pictures 5 inches apart.





ote formation of vortices. This is not a Karman vortex street. ight white swirl; others as dark spots.

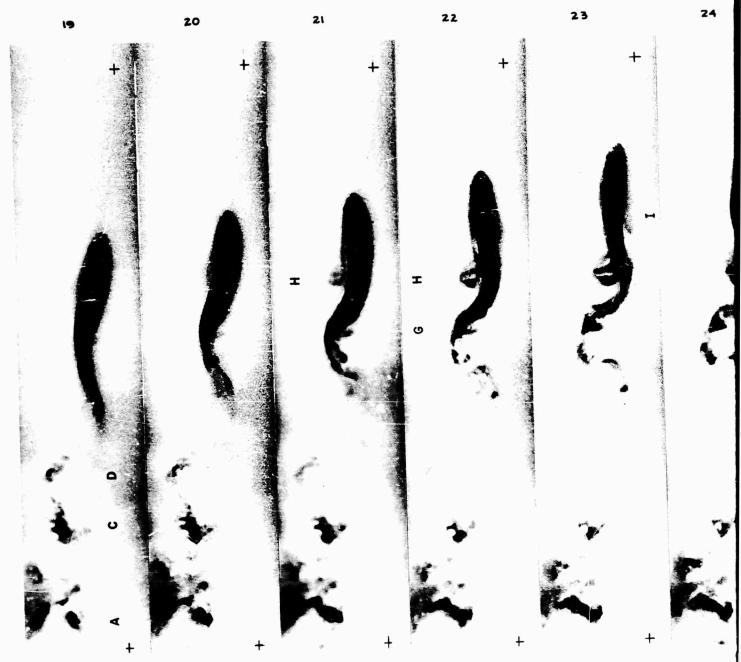
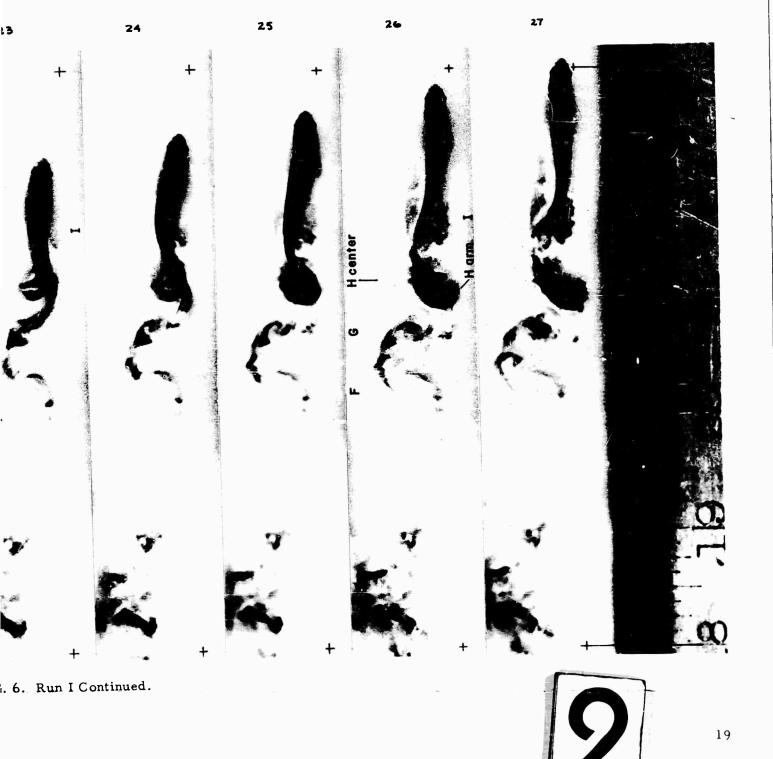
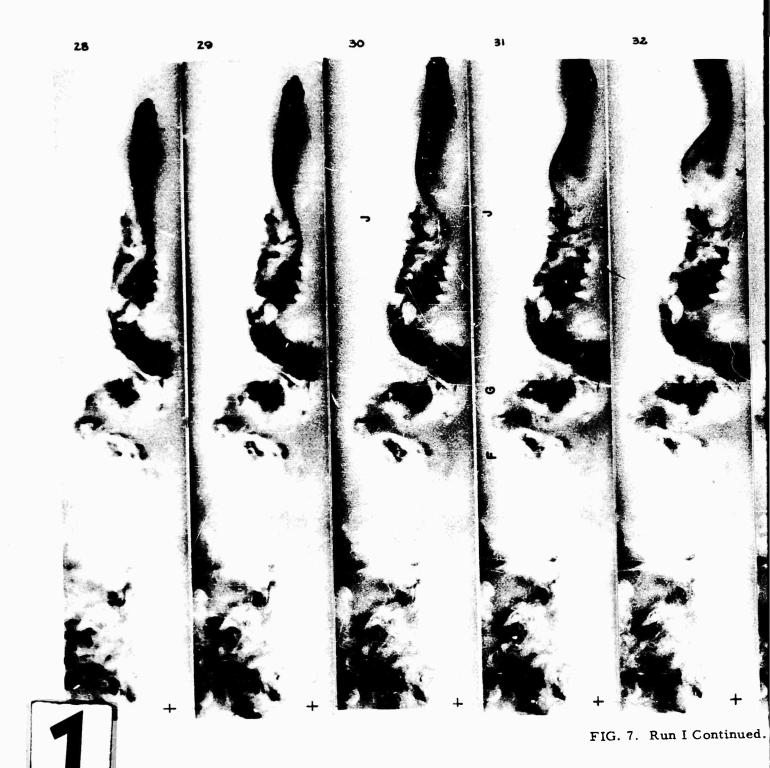


FIG. 6. Run I Continued.

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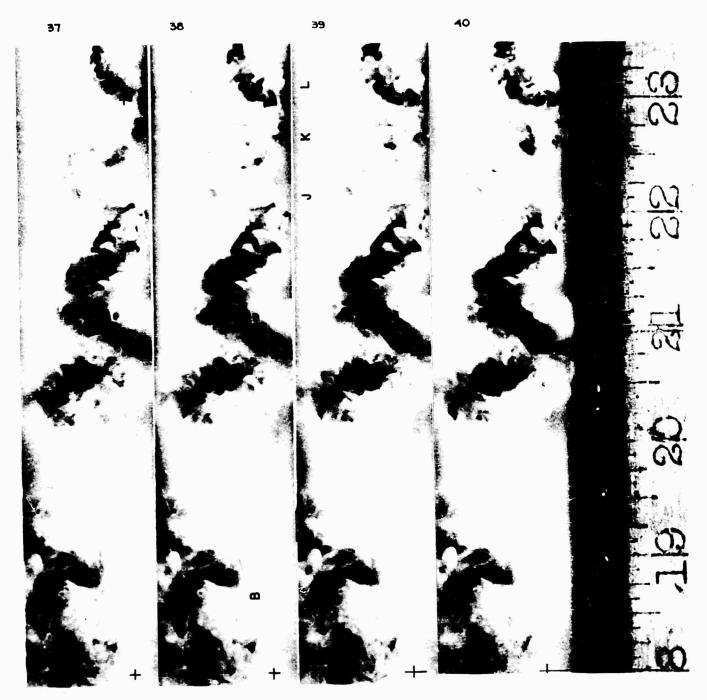


FIG. 8. Run I After Passage of Fish.

size. The body then deflects itself away from the vortex as it grows, so that it appears that the concave side of each flexural wave on the fish contains one vortex. Vortex C in Frame 13 (Fig. 5) is a good example of this; it is born in Frames 10 and 11. Vortices H and I in Frames 20 and 21 (Fig. 6) are other excellent examples.

### THE VORTEX PEG HYPOTHESIS

An action now takes place which to the writer is the most fascinating of all. The after side of the wave on the fish overtakes the vortex, seeming to press against it in the rearward and transverse directions; at the same time the skin is gliding alongside the vortex in the forward direction. Eventually the peduncle and the caudal fin (tail) pass alongside and forward of the vortex, engaging it fully and seeming to thrust upon it as if to propel it backward. However, the vortex exhibits no rearward acceleration at this point. The mechanism is as if these vortices were nails or pegs affixed to the ground, the fish reacting and thrusting against these stationary pegs with the after half of its body and then its tail. Thus, the fish, with the forward half of his form, first generates a regular and orderly system of pegs. Then, its body feeling these vortices by the sensing of pressures, it weaves its flexible form in and out between them, and reacts against them to produce forward thrust and propulsion.

This mechanism of action, hitherto unknown, is postulated by the writer as the Vortex Peg Hypothesis.

In an article by Gray (Ref. 5), an experiment is shown in which a trout, taken out of the water and placed on a dry board, wriggles but makes no forward progress. However, when taken out of water and placed on a board on which pegs have been nailed in a regular square pattern, the fish pushes against the pegs and moves across the board. In Fig. 9 one of Gray's photographs of the dry peg board and trout is shown and compared with the high-speed photograph of the writer's experiments in which a fish is swimming in water. The resemblance of action is striking. The dark spots are the vortices, the true pegs; the reaction of the tail against one of them is clearly seen.

It is possible, on a physical basis, to support this hypothesis and the existence of forces between the fish and the vortices. This subject will be discussed later.

### OBSERVATIONS OF RUN II, WITH SENSITIVE MILK LAYER

In Run II, the fish swam more leisurely at a speed of 7.8 to 11.5 in/sec over the sensitive milk layer. The sequence of oriented photographs is shown in Fig. 10 through 15. Although the speed is less than half, the pattern of vortices resulting is identical in form to that of Run I, as is the interaction between them and the fish. They are possibly more easily seen than in Run I.

Note again the lack of axial movement of the centers after fish passage. The eye can easily discern the single line in which Vortices G, H, I, J, K, and L have arranged themselves in Frame 61 (Fig. 13), each one revolving in the opposite direction (as a system of gears all in one line, meshing with each other).

Note that each stroke of the tail, and each undulation of the after half of the body, engages a vortex--seeming to react against it, as in Run I.

Frame 59 of this run is that used in Fig. 9 for comparison with the trout on a dry peg board. Vortex L in Frames 70 through 74 (Fig. 14 and 15) is another beautiful spiral galaxy with two arms.

### OBSERVATIONS OF RUN III, WITH DYE

This is the experimental sequence in which the fish was immersed in methylene blue dye, and then swam in clear water across the pulsing illumination coming up through the grid. The transfer of the fish from the dye to the clear water was accomplished manually with considerable speed, and the dye clung to the frightened darting animal for only about 1/2 second. Fortunately, the machinery of the camera and lamps was running and stabilized at that instant. Figures 16 and 17 show this sequence.

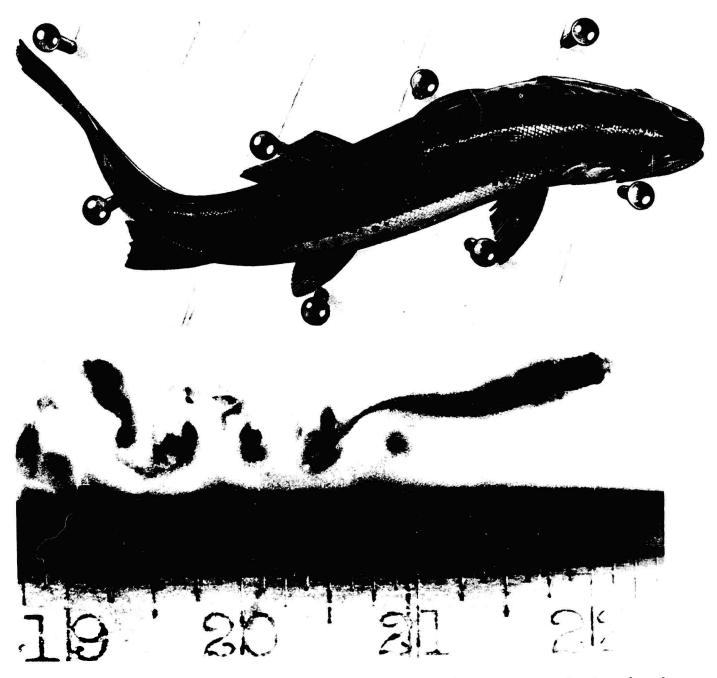


FIG. 9. Above: Experiment by Sir James Gray (Ref. 5). A trout on a dry board pushes against pegs and moves across board. Below: Experiment by author. Water flow is made visible about a small swimming fish. Vortices generated by fish are "pegs" against which body and tail react to produce propulsive force.

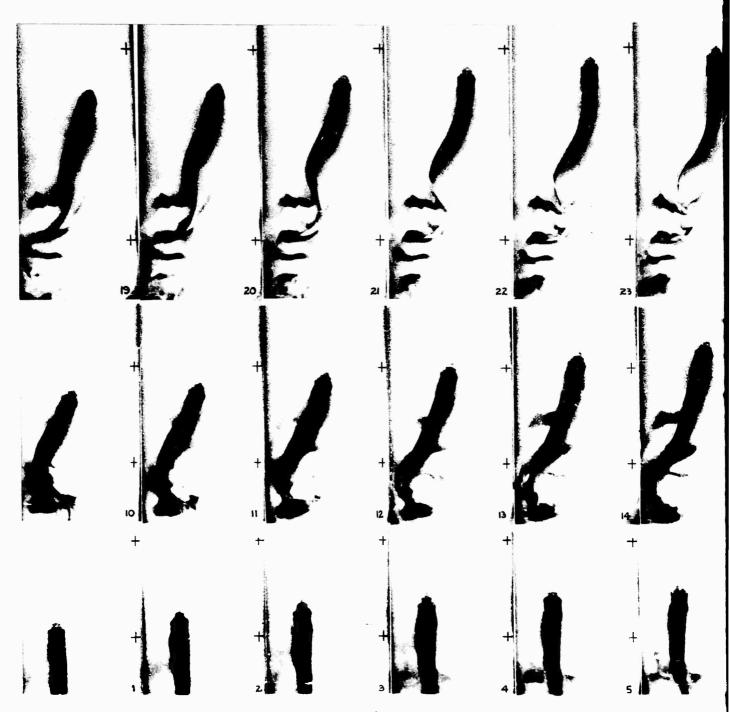
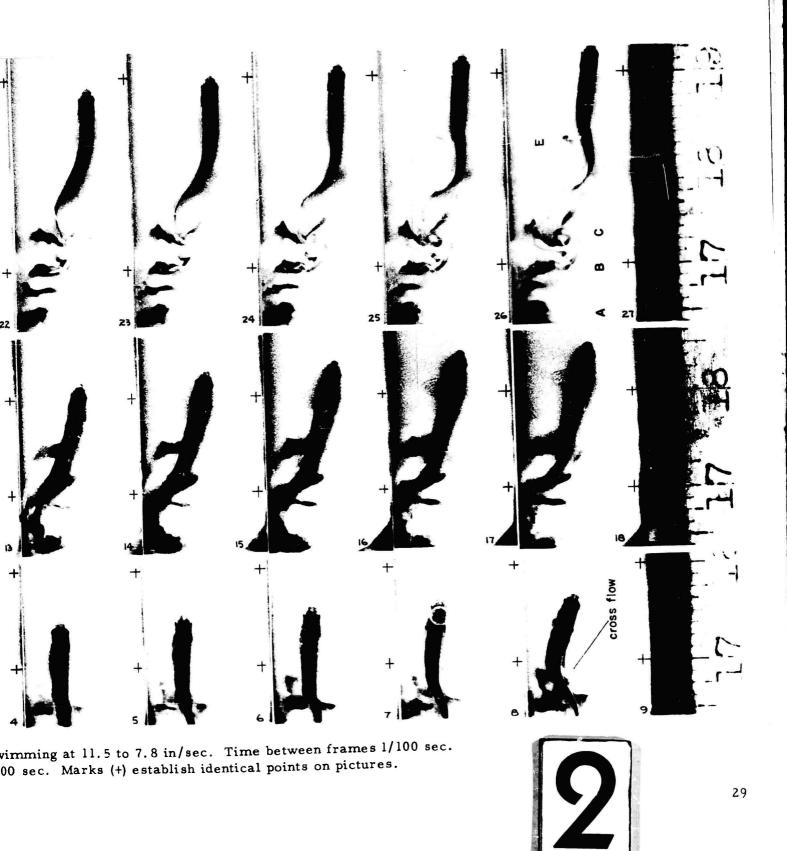
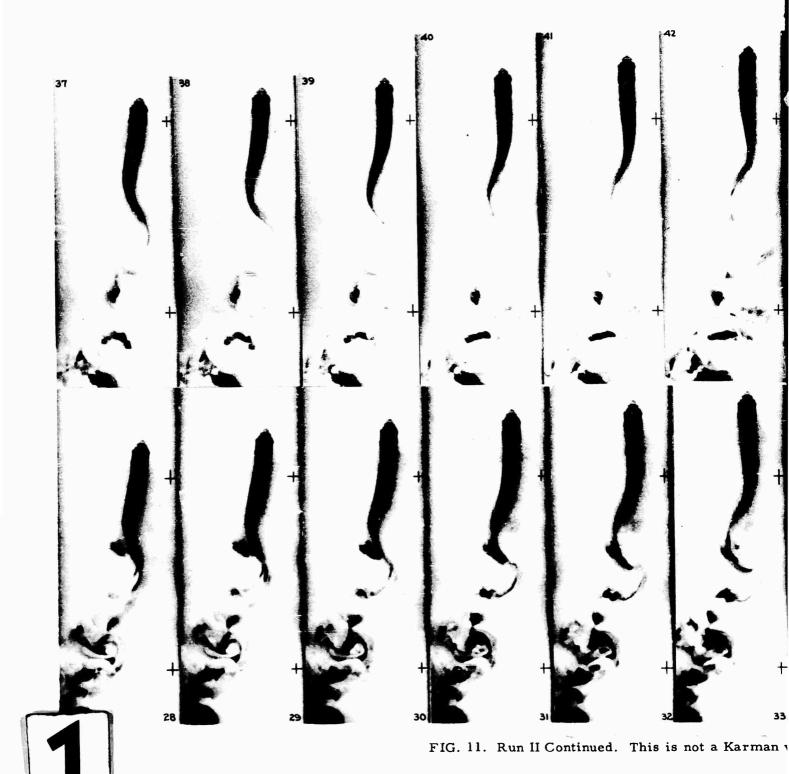
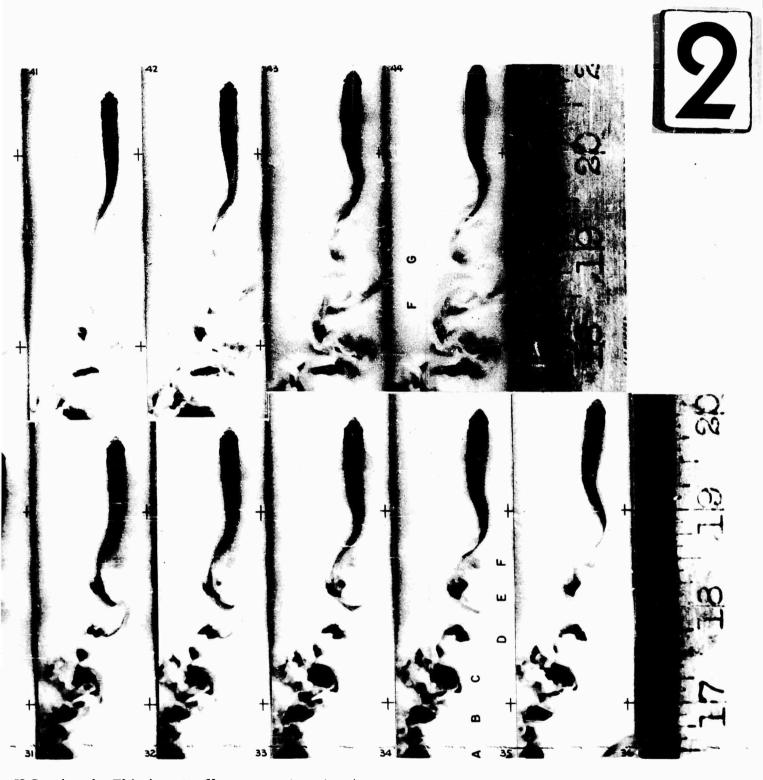


FIG. 10. Run II. Fish swimming at 11.5 to 7.8 in/sec. T Exposure time 2/1,000,000 sec. Marks (+) establish ident

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n II Continued. This is not a Karman vortex street.

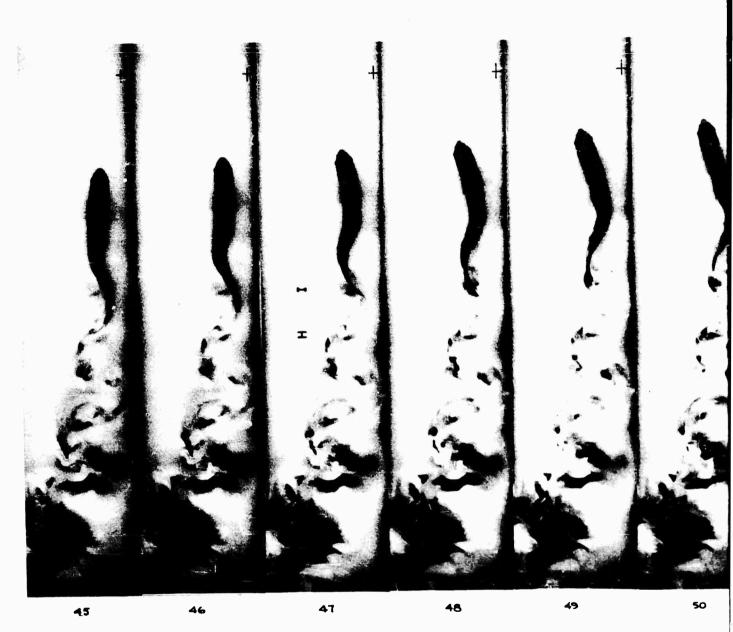
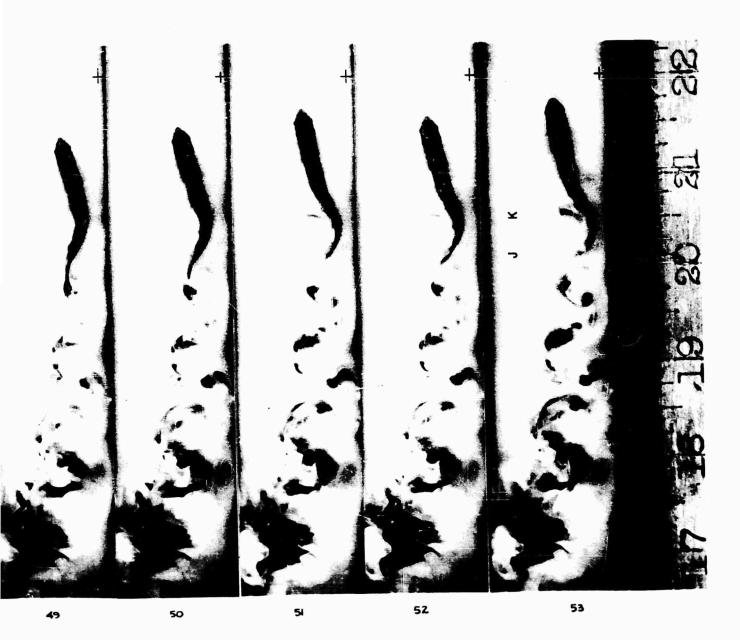


FIG. 12. Run II Continued. Note birth of ind

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I Continued. Note birth of individual vortices.

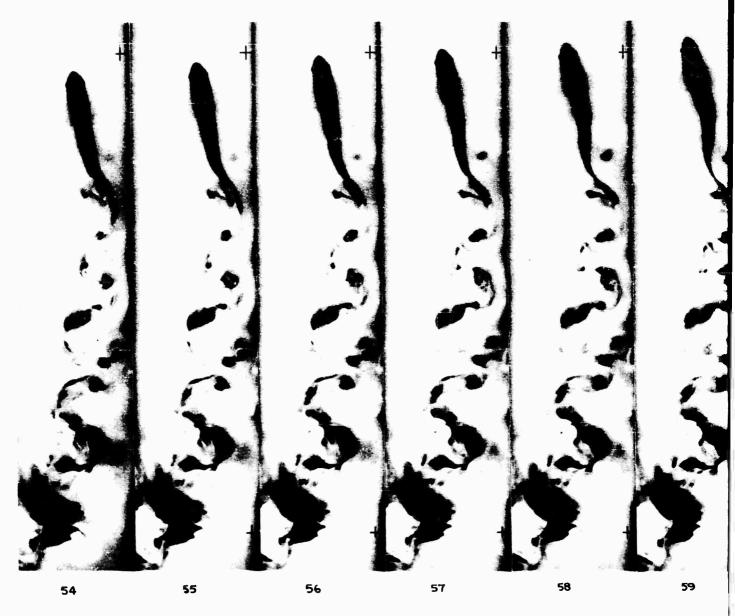
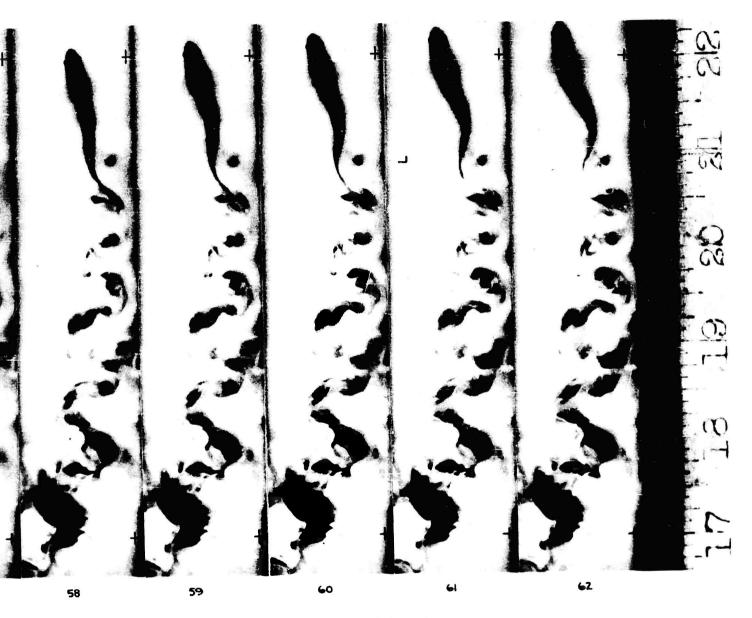


FIG. 13. Run II Continued. Note birth of Vortex L. Note lac of vortex centers and that they are in line with fish path rath

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Note birth of Vortex L. Note lack of appreciable axial motion they are in line with fish path rather than alternating.

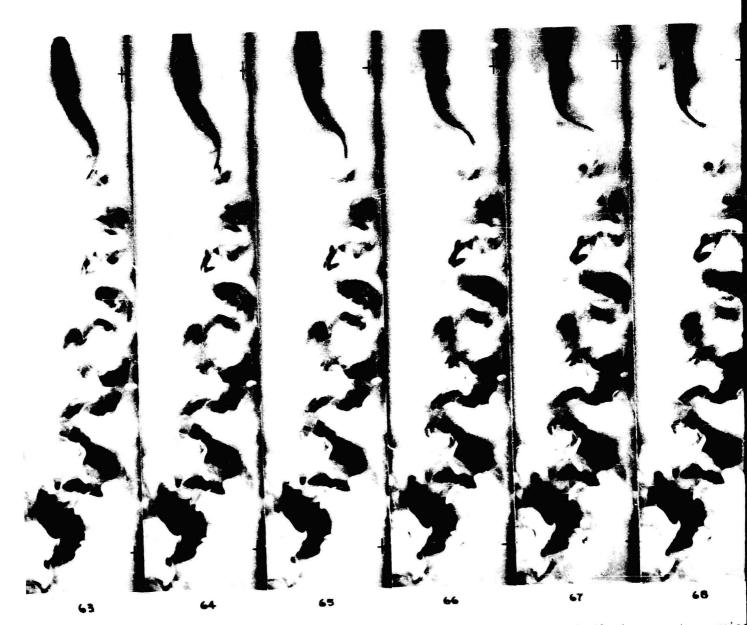
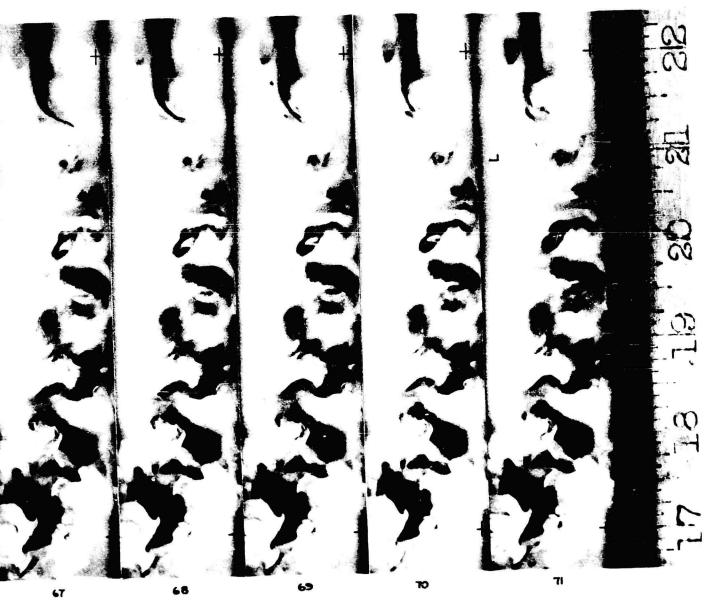


FIG. 14. Run II Continued. Note Swirl L similarity to astronomica





Swirl L similarity to astronomical spiral star nebulae (galaxies).

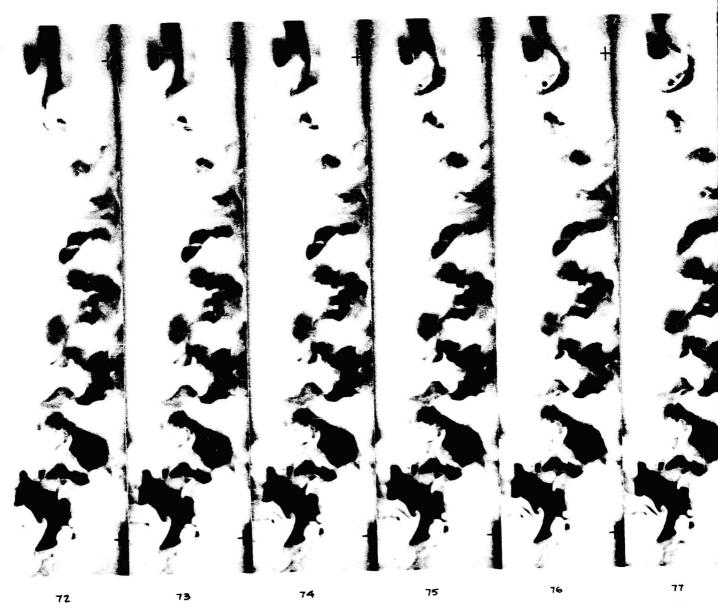


FIG. 15. Run II Continued. Bars are arms of spent vortices.



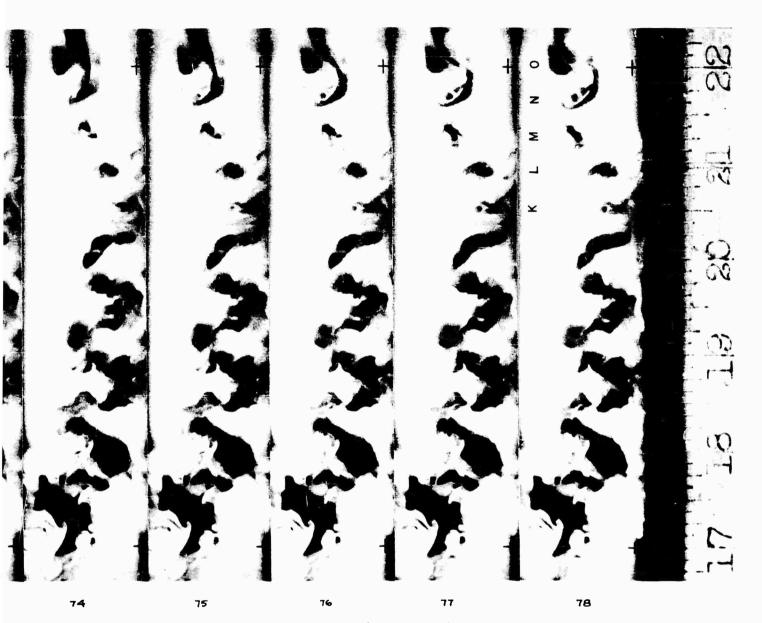


FIG. 15. Run II Continued. Bars are arms of spent vortices.

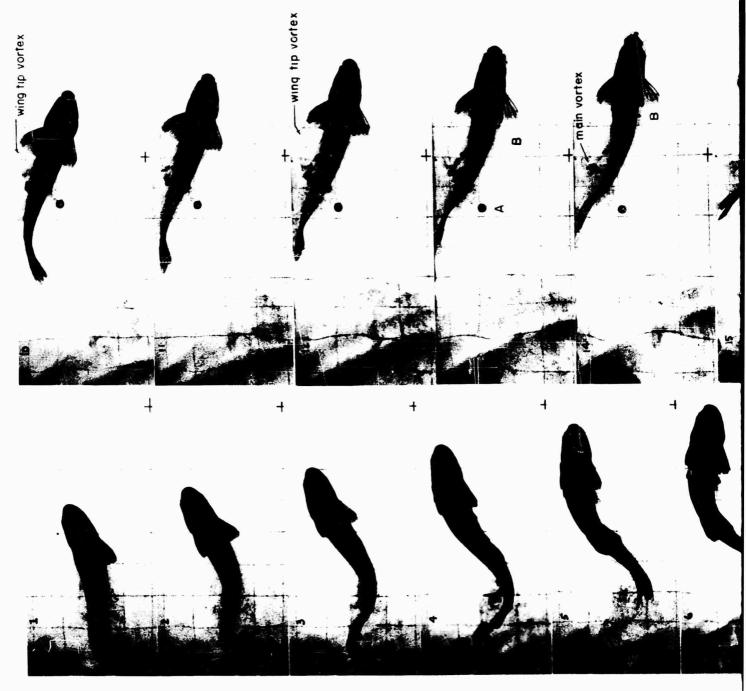
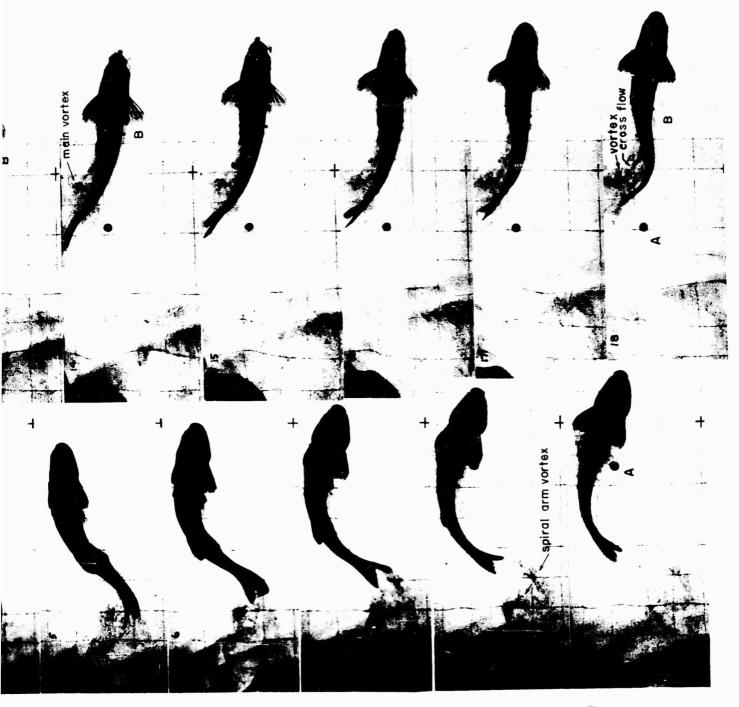
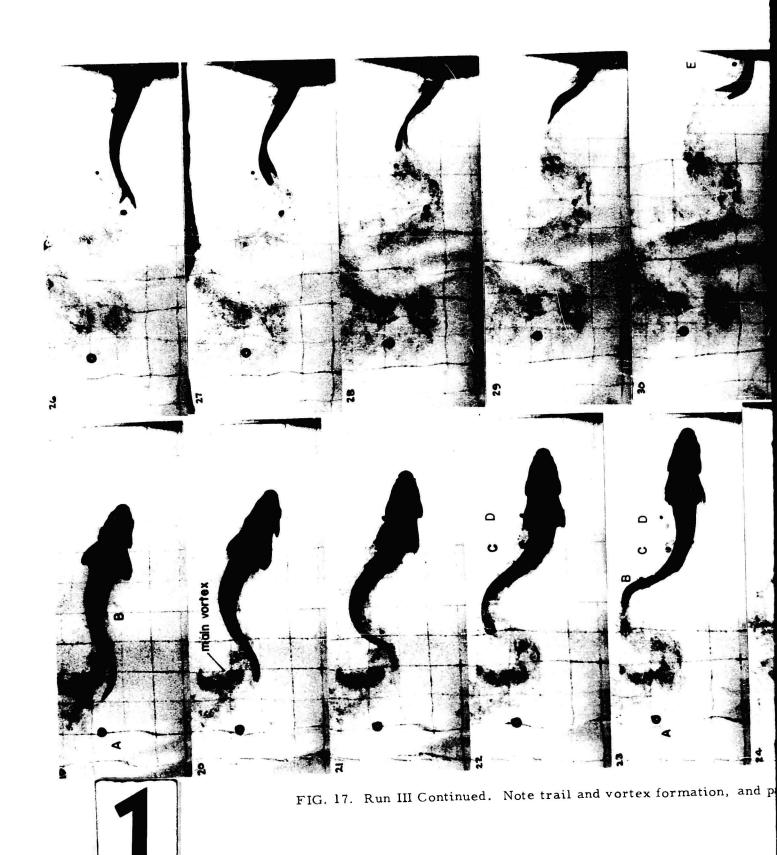


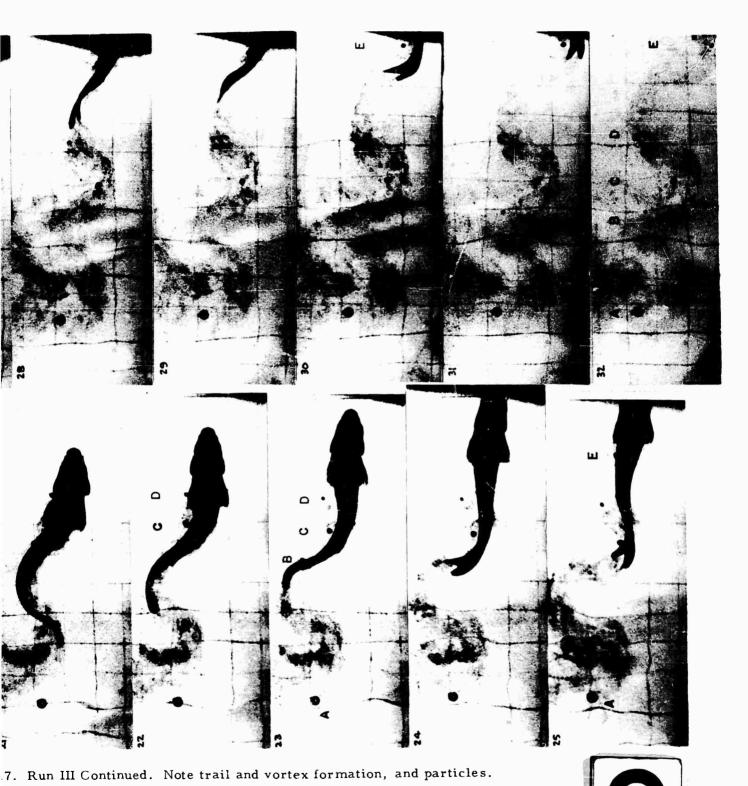
FIG. 16. Run III. Methylene blue and particles flowing off fish. Exposure time 2/1,000,000 sec. Maximum speed 22 in/sec. Gr





ne and particles flowing off fish. Time between frames 1/100 sec. Maximum speed 22 in/sec. Grid spacing 0.25 inch.





The trail of dye seems at first glance to be different in character from that left with the sensitive quiescent milk layer. A well-defined, continuous trail which is wavy but not sinusoidal is traced by the tail. It should be observed that dye will mark only those layers of water which have flowed close to the skin of the fish, the boundary layer. The quiescent milk shows not only the boundary layer but the entire field of flow disturbance to some distance on either side of the fish. Thus the dye pattern, even in theory, should appear different from the milk pattern. However, with careful study, the dye is seen displaying a portion of the same basic phenomenon as was shown by the sensitive milk.

The wavy trail actually consists of the connecting arms of the main, in-line, contrarotating vortices. It can also be observed that the trail has bumps or "knots" in it, which are approximately in line with the fish's path. These knots are the vortex centers, and the circular flow lines in them can be made out. One of these vortices, after the fish's passage, developed into beautiful galactic spiral nebulae of dye, with two graceful arms (Frames 8 and 9 in Fig. 16). The waves in the trail do not resemble the mathematical sine curve, but are actually discontinuous.

#### THE CROSS FLOW

A phenomenon of great interest, only suspected in Runs I and II, is now proven to exist. On Frames 17 and 18 (Fig. 16) a dark filament of dye can be clearly seen in which the water is flowing from the edge of the fish crossing to the other side. In addition, another dramatic action is shown: this cross-flow filament can be discerned in Fig. 17 feeding directly into the center of a main vortex.

It is seen that the character of the entire field of flow is at the same time both complex and yet very systematic and orderly. It does not resemble laminar or streamline flow as did the previous concept. Nor does it resemble turbulent disorderly flow. It is perhaps a new type of fluid motion system in which the body and the fluid processes are intimately interwoven with each other, their interaction regularly producing and then using for propulsion discrete unit quanta or "flow packages." And unlike manmade devices such as ships or planes which produce propulsion

by driving a stream of fluid rearward (propellers, jets), the fish, unless it is accelerating, creates no net backward-moving stream of water.

Out of all the unorthodox actions which the water and fish are performing together, one event is observed with which we are familiar. Streaming from the tip of Geronimo's little left fin on Frames 10 and 12 (Fig. 16) is a tiny wavy filament. This is the well-known wing-tip vortex which aerodynamic action produces on the ends of aircraft lifting surfaces such as wings.

Run III also provided, purely by accident, a lucky addition to the data. Round particles, of unknown composition but obviously impregnated with the dye, issued from the mouth of the fish as he swam. These were swept back into the flow around the fish. Their motions were studied and will be discussed later.

## SECONDARY VORTICES

One of the earliest series of photographs, taken at the writer's home, was an 8-mm color movie of the small fish swimming over the sensitive milk layer. Figure 18 shows an enlargement of one of these films. This picture was selected because, for some reason, the small secondary whirls which are at the end of the arms of the main vortices were the first to appear. They form a zigzag pattern as at the corners of a series of 60° equilateral triangles. Slightly farther back on the trail these disappear and the main row of vortices makes its appearance in a single straight line.

## FLUORESCEIN-DYE EXPERIMENTS

In another early experiment, a small fish was immersed in a solution of fluorescein sodium and then photographed in sunlight swimming down the black channel. This dye left a trail that glowed green against the black. Figure 19 is an enlargement made from the very small 8-mm picture. The wavy trail is similar in shape to that obtained later in Run III with methylene blue dye. The reader may discern the knots in the middle of each wave. These knots are the centers of the main vortices, the rest

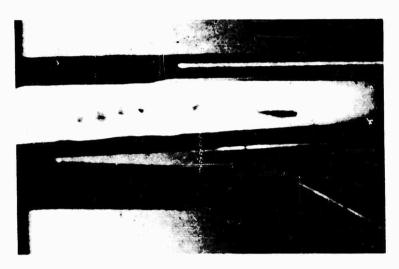


FIG. 18. Zigzag Secondary Vortices. Main swirls appeared later, in line.



FIG. 19. Fluoroscein Sodium Trail. Vortex centers (knots) in line.

of the trail being the vortex arms. Note that these knots are in line with the path of the fish.

Fluorescein sodium unfortunately is quite toxic to fish. This little performer died. In one of the writer's subsequent experiments with a 41-inch Pacific yellowtail kindly furnished by Marineland of the Pacific, it was thought that this large strong fish could withstand a small amount of fluorescein. However, this dye weakened it at once so that it did not swim normally, and died within an hour.

### DOLPHIN EXPERIMENT

In the work with the small fish, the Reynolds number of the action was quite low, computed to be

$$= 1.36 \times 10^4$$
 at 11.5 in/sec

The swimming of a dolphin occurs at much higher Reynolds numbers ( $R = 1.55 \times 10^7$ ). It was felt that a worth-while comparative experiment would be to attempt the visualization of the flow about this intelligent and graceful creature, whose caudal fin lies in a horizontal plane and moves up and down, since it is a cetacean.

It was reasoned by the writer that as a dolphin leaps out of the sea into the air, a portion of the water in the boundary layer next to its body is carried with it. It might be possible that the lines of flow would then be visible.

Figure 20 is a photograph taken by the writer just as a dolphin emerged from the water surface at a measured speed of 18.5 mph in an easy leap at Marineland, California. This leap was a rather slow one--there were much faster exits but they did not show the flow lines as clearly. This shot, exactly broadside to the dolphin, required much patience to obtain and involved many failures.

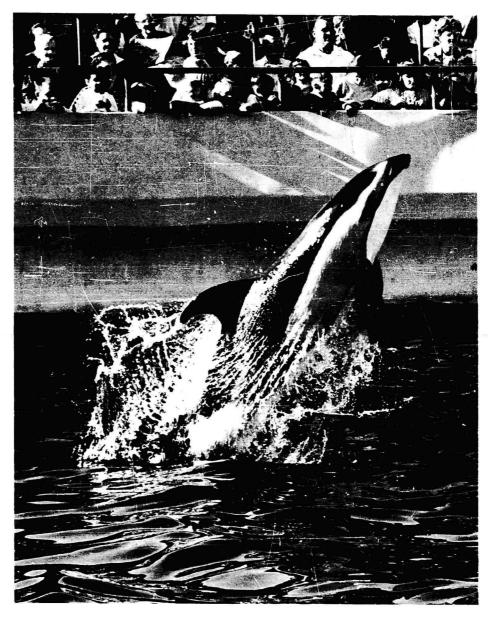


FIG. 20. High-Speed Photograph of Pacific White-Sided Dolphin Leaving Water at Measured 18.5 mph at Marineland, Calif. Dolphins can jump faster. Character of water flow revealed (see Fig. 22a).

The flow lines are indeed visible. It is believed that some (not all) of the distinct water lines seen are long, narrow, twisting filaments or vortex cores. But the truly startling occurrence is that these filaments, including those on the side and those being shed by the dorsal fin (on the dolphin's back), are bent into concentric ellipses, the center of these ellipses located temporarily at the dorsal fin. Could these be the flow lines of a main vortex, beginning to form at the midbody of the dolphin?

Figure 21 shows the state 0.17 second after Fig. 20. The water has been formed into the wavy trail now familiar to us from the experiments with the small fish. The dolphin has done this work with a sweep of its tail and two thirds of its body, flexing from a point at its lower forward fins. The dolphin is still "swimming" although in air! In addition, two main vortex cores may be seen. One is reasonably clear as the knot in the center of the upper wave, the remainder of this vortex being its two arms. Is this the elliptical vortex in Fig. 20 after being operated on by the dolphin? The lower vortex is the heavy knot of water just above the surface, not quite so easily made out. The arms of the upper vortex are tipped by two less dense secondary vortices, with very clear circular patterns. The picture is completed by a cross-flow stream which is quite clear, breaking away from the lower body, probably at a steeper angle than normal because of gravity.

To assist in identifying these flow features, Fig. 22a and 22b have been prepared. The lower main vortex is difficult to see without very close examination of the curving lines in the white foaming water.

The entire flow picture is startling in the similarity of its basic design to that of the small fish. The writer is forced to conclude that despite the large difference of Reynolds number, both the small fish and the dolphin create the single-row Vortex Peg System about them as they swim.

The lower front fins of the dolphin are not used in propulsion, but probably in steering. It is interesting to note in Fig. 20 that these fins are shedding water sheets which do not have a well-defined pattern of filaments as does the water on top of the dolphin. It is reasoned that this may be because the portion of its body

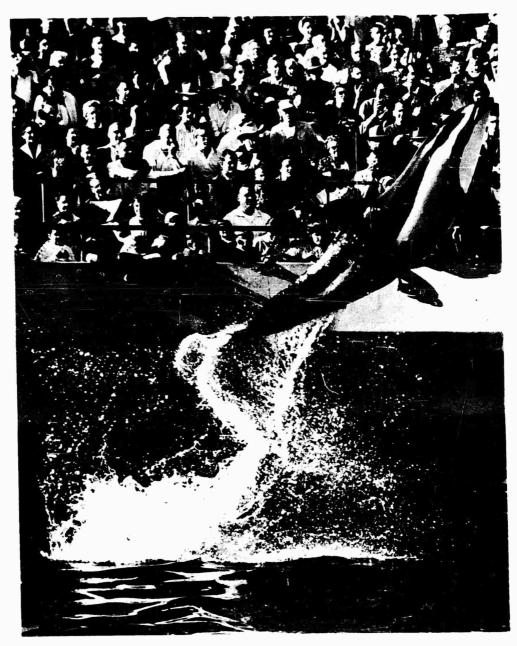


FIG. 21. The Dolphin 0.17 sec After Fig. 20, Still "Swimming" and Generating Vortex Pegs and a Trail Similar to That of Small Fish Experiment (See Fig. 22b).

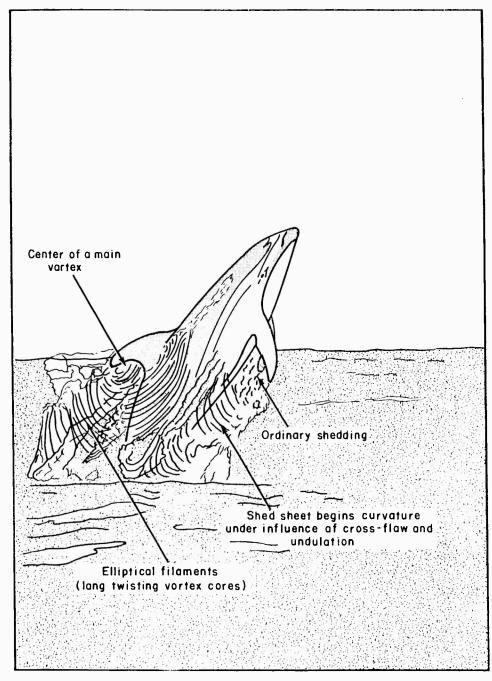


FIG. 22a. Explanatory Drawing for Fig. 20.

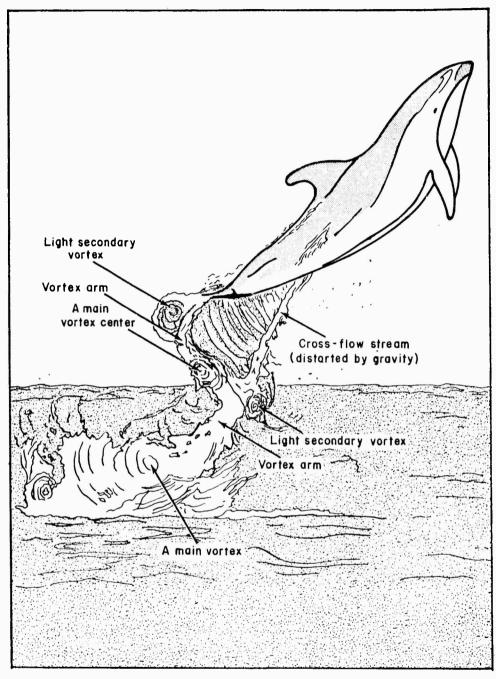


FIG. 22b. Explanatory Drawing for Fig. 21.

forward of the lower fins does not flex to engage in propulsive effort. In other words, it is believed that the curved elliptical filaments are not merely the result of ordinary boundary-layer flow such as is presently known over rigid objects, but is the result of the flexure of the surfaces and the freedom for upward or downward cross-flow to occur. The body ahead of the lower fins does not undulate, hence ordinary flow is shed by them. The body ahead of the dorsal fin does undulate, hence the flow is transformed.

Thus, in its vigorous and playful way, the dolphin in its leaps has for thousands of years been revealing to us the manner in which it swims. We have only to observe with a clear and patient eye the ordinary yet wonderful events in nature.

### IDEALIZED VORTEX PATTERN

The flow pattern of vortices after the fish has passed, from the evidence of the experiments, has been sketched in an idealized form in Fig. 23. Here the animal is assumed to be swimming from left to right in a straight line, leaving behind a series of "gears" all in line. The path of the tail is the dotted wave which traverses a course just in front of each main vortex. The arms of the main vortices link themselves together by means of the small secondary curls S, and these arms form the wavy trail which appears when dye is used as in Run III.

The small linking vortices S form a 60° triangular pattern which makes its appearance in Fig. 18 over the sensitive milk layer.

When the main vortices are first born, just aft of the head of the fish, they are not in line, since they must be on one side of the fish or the other. But the body's undulatory motion moves them in a direction perpendicular to the path of the fish so that they fall into line usually before passage of the tail.

The flow of water between the main swirls at the center line is perpendicular to the fish's path of motion. This is where the circulation of two adjacent vortices combines.

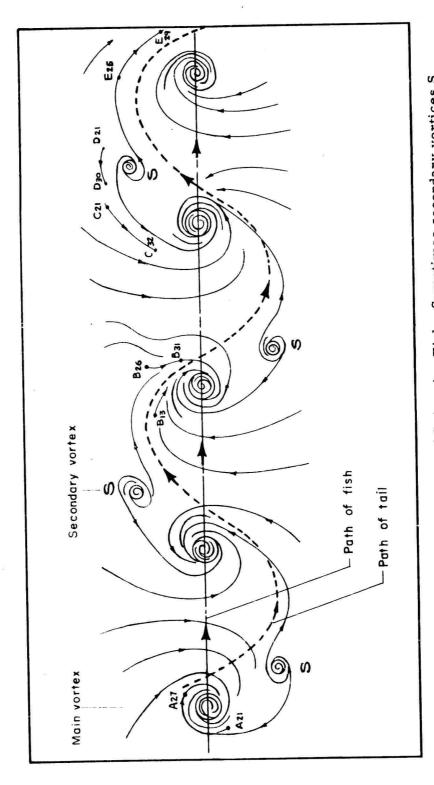


FIG. 23. Idealized Vortex Pattern of Swimming Fish. Sometimes secondary vortices S are seen first in zigzag pattern, the main vortices appearing later (Fig. 18). This is not a Karman vortex street. Particles A, B, C, D, and E of Run III are shown in probable relation to flow (A21 is Particle A of Frame 21, Run III, etc.).

After the animal has passed, it is believed that the speed of the fluid in these vortices is quite low in comparison to the velocity of the fish itself.

# CONTINUOUS CHARTS OF MOTION, RUNS I AND II

Two charts, Fig. 24 and 25, were made by accurately tracing all the individual photographs of each run on one piece of paper. The actual paths of the nose and the tail tip are shown in 1/100-second intervals; the location of each vortex center was also located by direct tracing. The form of the fish is traced in its true shape and position in several places. Each nose and tail position is located by numbered points (22n = nose in Frame 22, and 19t = tail in Frame 19). Each vortex is identified by letter.

These integrated charts give a good picture of the propulsive action as postulated by the author's Vortex Peg Hypothesis. The path of the tail is almost always obliquely in front of each vortex. Each concave undulation of the fish's body contains one vortex which is generated well up front, grows in size, is reacted upon by the rear side of an undulation of the fish's body, and is finally reacted upon by its tail. The physical intricacies of this reaction will be discussed later, and the theory of screamlines of fluid motion about the body of a swimming fish will then be proposed.

The two charts can be measured (within ±0.02 inch) for computation of velocities, accelerations, and paths. The stadia marks (+) correspond with those on the photographs.

## MOTION OF PARTICLES

A fortunate accident occurred during the dye experiment of Run III. Five spherical particles appeared in the flow--close to the fish. One of them is seen coming from the mouth in Frames 14 and 15 (Fig. 16), apparently impregnated with the dye.

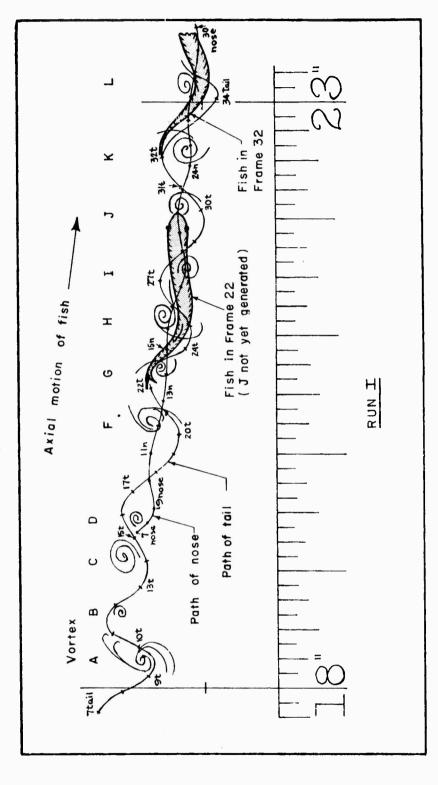


FIG. 24. Continuous Tracing of Sequence Photographs of Fish Swimming at 24.0 to 18.8 in/sec. Time Vortex centers have little or no axial motion after fish passage. This is not a Karman vortex street. between marks 1/100 sec. Note locations of vortex on body, and how they line up with nose path.

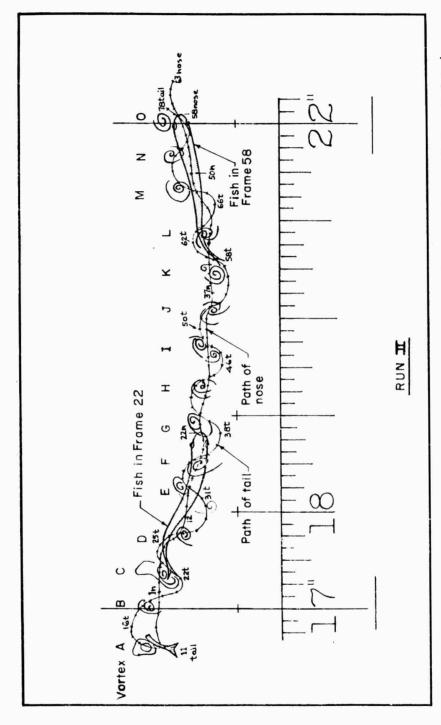


FIG. 25. Run II, Continuous Tracing of Photographs of Fish Swimming at 11.5 to 7.8 in/sec. Time between marks 1/100 sec. This is not a Karman vortex street.

The motion of these particles was studied by the continuous tracing method, the results appearing in Fig. 26. The location of

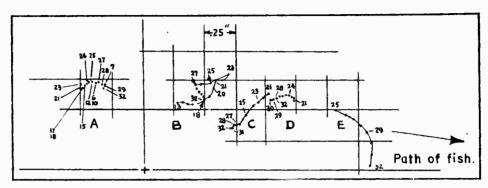


FIG. 26. Motion of Particles. Traced from photos of Run III. Most particles are brought almost to a halt.

each particle was determined from the grid lines nearest to the particle. Thus distortion of the lines due to gentle ripples (caused by transferring the fish to the channel) was largely cancelled out. The grid lines are spaced at 0.25 inch (±0.005). The history of these particles as they sweep around the fish is as follows:

Particle A Drifts slowly forward, backward, forward, comes almost to a dead stop.

Particle B Drifts slowly forward, accelerates to 6 in/sec obliquely to left, suddenly reverses to aft 8 in/sec, reverses again and almost halts.
(B clings close to fish's right side, is caught in cross-flow, changing to left side. Further proof of cross-flow.)

Particle C Straight rapid movement obliquely aft and to right at  $ll^{\frac{1}{2}}$  in/sec. Suddenly almost halts.

Particle D Very slow drift to aft, reverses to forward slowly, almost halts.

Particle E Moves forward, curves smoothly to right and assumes course perpendicular to fish's motion. Subsequent history not seen.

It is evident that the motions of these particles are not the same. Some move forward, some move aft, four travel most decisively in directions perpendicular to the path of the fish. Several significant facts may be observed:

- 1. There is no evidence of a rearward-moving current.
- 2. There is no evidence of a forward-moving current.
- 3. Some particles are initially accelerated forward in the boundary layer to 38 to 50% of the fish's 16 to 22 in/sec velocity, then are suddenly brought almost to a halt, remaining with a very small, random wobbling motion.

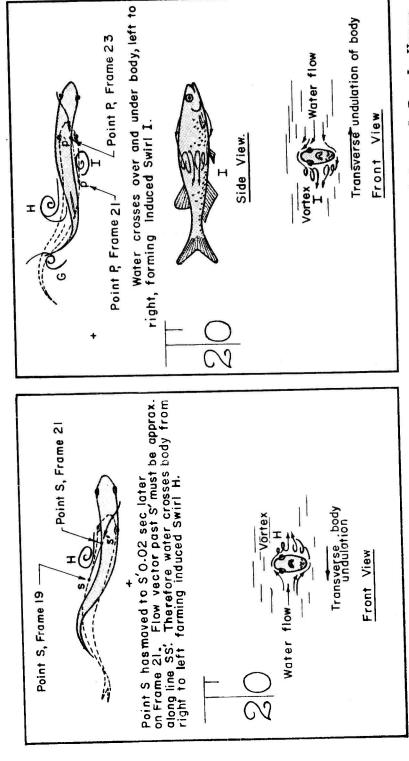
## HOW ARE THE VORTICES PRODUCED?

In watching the birth of the vortices, a natural question arises. Where do they come from?

The explanation can be found. In Fig. 27 and 28 (Run I) careful tracings have been made of the fish's shape at intervals of 0.02 second, then accurately superimposed on each other.

In Fig. 27 a point S is selected on the fish of Frame 19 (Fig. 6). Point S is located again on the fish of Frame 21 which is 0.02 second later. Connect these two locations by line S'S. It is assumed that a particle of water near the lower edge of the fish had an approximate motion (relative to the fish) roughly in the direction of this line. This makes the simple assumption, only partially correct, that the particle is not moving with respect to the ground. The direction of line S'S shows that the twisting-forward motion of the front part of the body should force fluid particles to cross the body obliquely both above and below. Since the water crosses a somewhat flattened shape with a component normal to the plane of flattening, a double vortex or swirl is formed on the downstream side.

The lower sketch of Fig. 27 shows this action, giving rise to an induced Vortex H.



L Point P. Frame 23

FIG. 27. Production of Vortex H, Run I. Upper drawing is exact tracing from photograph.

FIG. 28. Production of Vortex I, Run I. Upper drawing is exact tracing from photograph.

Transverse undulation of body

Front View

-Water flow

Side View.

In Fig. 28 the same method was used with photographs of Frames 21 and 23 of Run I (Fig. 6). The water flow past Point P is such that the cross-flow is reversed, creating Vortex I. The side-view drawing can perhaps best illustrate the water path. This entire action is much aided by the twisting of the head from side to side with each undulation.

## STRUCTURE OF VORTEX

The writer henceforth desires to present a concept which on a physical basis can explain the Vortex Peg Hypothesis earlier presented.

The basis of reasoning is taken from the photographic evidence itself. In Fig. 29, the image of the fish from Frame 22, Run I (Fig. 6), was photographically enlarged. Its exact shape was then traced; also the vortices seen in that picture were traced as closely as possible. These are

- I A newly born vortex
- H A developed, energetic vortex
- G An adult vortex (G on the photo is small and tight, hard to distinguish)

Notice the position of these swirls with respect to the body.

## Vortex Necklace

Figure 30 is a drawing of the side view based on the actual enlargement in Fig. 29. This shows the probable placement of the vortices with respect to the fish and its tail. The swirls of Vortex H are dotted because they are on the other side of the fish. The after cross-flow filaments, known to exist, are shown. The initial cross-flow filament has been proposed as responsible for vortex birth. Another mid-body filament is now proposed, connecting I and H. If this filament also exists, the fish is embraced in a beautiful "vortex necklace" through which it winds in and out.

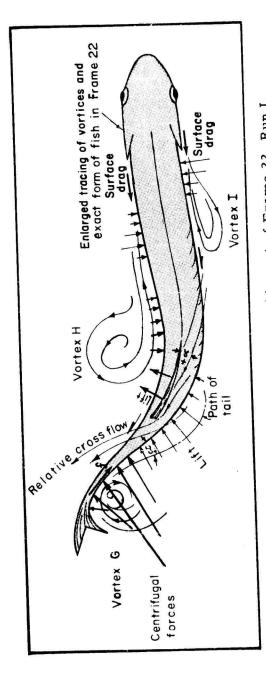


FIG. 29. Forces Acting on Fish at Moment of Frame 22, Run I.

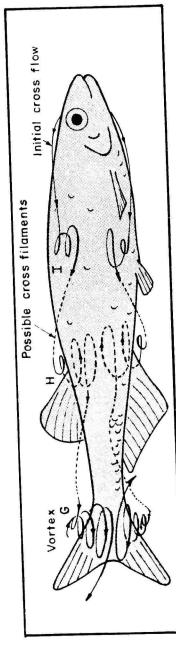


FIG. 30. Vortices in Side View, Showing Possibility of a Connected Vortex Necklace System.

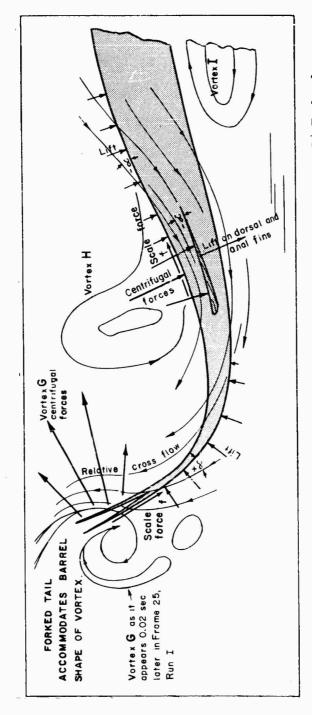


FIG. 31. Relative Streamlines and Forces of Moment, Frame 23, Run I. Body shape (Frame 22, taken 0.01 sec previously). Angle of attack α on dorsal and anal fins has and vortices traced from enlargement of photograph. Note changes from Fig. 29 reversed. Vortex H has begun expansion.

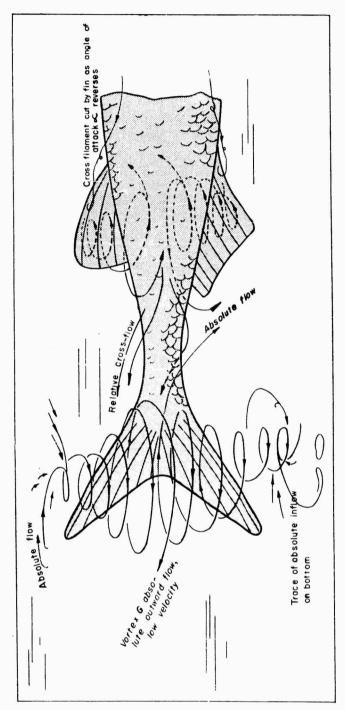


FIG. 32. Possible Barrel Structure of Vortices, Spiraling Toward Fish Center. (Corresponds to Fig. 31.)

Assuming symmetry of flow, the vortex structure must be a double swirl, as shown--with the sources feeding into the center.

The state 1/100 second later is shown in Fig. 31, which is a greatly enlarged tracing of the actual fish and vortices in Frame 23, Run I. Figure 32 is a drawing based on the actual conditions of Fig. 31. Figure 32 shows in detail what is believed to be a barrel-shaped structure for each vortex--thought of as a double symmetrical swirl. The feed source is at the two ends, with the flow swirling inward toward the center line of the fish in widening spirals. At the center line, the two flows meet and then leave the vortex in large, low-velocity spiral paths.

Figure 33 shows the possible net flow pattern for the barrel vortex. The axis of rotation is vertical.

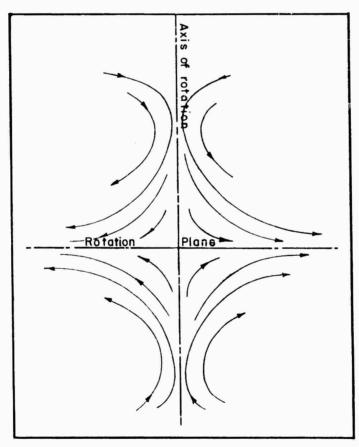


FIG. 33. Net Flow Pattern of a Barrel Vortex.

#### VORTEX FORCES ACTING ON FISH

With the foregoing picture in mind of the manner in which the water moves within a vortex, it is now possible to understand that certain forces may exist.

Every mass travelling in a curve exerts a centrifugal force and tends to move away from the rotation axis. The centrifugal forces present in any vortex manifest themselves by a decrease of fluid pressure at the center and an increase at its periphery. At present, fluid-dynamics literature analyzes a vortex as having a total energy no different from the still water in which it finds itself. In this kind of a swirl, termed a "free vortex," the pressure at its periphery is the same as that of the surrounding still water. In a free vortex the relationship of the velocity V of an individual particle to its radius r from the rotation axis is

#### Vr = a constant

However, if a vortex is created by adding kinetic energy to the fluid by mechanical means, such as a rotating-vane system, then a "forced" vortex results in which

 $V = r_{(1)}$ 

where  $\omega$  is the constant angular velocity. In a forced vortex, the centrifugal forces are such as to create a pressure greater than ambient. This is the principle on which the centrifugal pump operates for the pumping of water.

In the case of the fish, it can be clearly seen that the surfaces of the fish come in contact with the vortices. It is the writer's hypothesis that these vortices are momentarily of the forced vortex type; and that they exert substantial forces on the flexural waves of the body, on the surfaces of the dorsal fin and the anal fin, and on the tail surfaces. These forces are purely and simply centrifugal in origin. Since the forward part of the fish has brought the vortex into being by adding energy to the water, the vortex that is created can temporarily develop a pressure at its periphery that is above the ambient.

The fish has the ability to flex its form in response to the pressures which the sensory nerves at the sides of its body must actually feel. The motor impulses governing the muscles cause them to bend the body so as to present to the centrifugal forces surfaces which are inclined. The centrifugal pressures can then develop a forward force component or thrust.

Figures 29 and 31, actual enlargements of the fish shape and the vortices, show how these forces radiating from the vortex centers create a forward thrust upon the body, upon the after fins, and upon the tail. The centrifugal forces are not to be thought of as the same thing as the normal aerodynamic lift forces.

The lift forces, first postulated by Gray as being caused by the tail sweeping the water at an angle of attack a, also create a forward-thrust component. Figure 29 shows that such lift forces are not only limited to the tail but can also be exerted by the body ahead of the dorsal fin, and by the dorsal and anal fins also--due to the cross-flow phenomena.

The centrifugal forces emanating from the vortices are forces which are in addition to the lift forces; and are believed to probably represent the major forces driving the fish forward.

#### SYNCHRONIZATION

An examination of Fig. 29 and 31 will show that the centrifugal forces from each vortex work in synchronization with the lift forces. To illustrate: in Fig. 29, the dorsal fin is not engaging Vortex H, but it is creating some lift and thrust due to the positive angle of attack +a with the cross-flow. One one-hundredth of a second later (Fig. 31), the dorsal fin has moved so that it has cut the upper cross-filament leading to Vortex H. At that instant this fin reverses its angle of attack to -a. At the same time the dorsal fin engages Vortex H, which exerts centrifugal force upon the fin as well as the body. As can be seen, the body has flexed so that the centrifugal forces on the dorsal fin now add to the aerodynamic lift from the cross-flow, and both have a forward component of thrust.

Vortex G, in both Fig. 29 and 31, is working away on the peduncle and tail which has wrapped itself around G so neatly, as if to confine it.

#### STATIC FORCE AND TORQUE BALANCE

The complete static balance of forces and torques can be seen in Fig. 29. The skin-drag forces on the front third of the fish's body are balanced by the thrust components of the centrifugal and lift forces. The head is twisting clockwise at this instant, transferring a clockwise torque to the water and giving rise to the birth of clockwise Vortex I on the right side (by virtue of the cross-flow).

The water pressures created by the twisting head are shown just aft of the gills on both sides. These pressures, together with the centrifugal forces from Vortex H, exert a counterclockwise torque on the fish's body. These equalize the clockwise torques from Vortex G on the tail and the lift on dorsal fin and rear body.

#### THE STREAMLINES ABOUT THE FISH

The method of superimposing two photographs can now be put to further use.

The fish's form of Frame 24, Run I (Fig. 6), was greatly enlarged and superimposed on Frame 23, taken 1/100 of a second earlier. Both forms are correctly located in the absolute system of coordinates.

Arbitrary points (1, 2, 3, 4, etc.) were chosen on the earlier form. They were then precisely located by measurement on the later form (1', 2', 3', 4', etc.). By connecting these points, and assuming that the fluid has little movement in the absolute system, a reasonably good picture of the flow lines can be determined. These streamlines are fluid paths of motion relative to the fish's form. See Fig. 34.

Figure 35 was then almost automatically derived from Fig. 34 to give a complete picture of the field of streamlined flow relative to the fish and vortices of Frame 24.

In this streamline picture the relative flow lines at times have directions opposed to the absolute flow lines in the vortices. There is no conflict, since a particle may be travelling backward relative to the fish's motion, while its own motion may actually be forward with respect to the ground.

The important features to observe in this rather extraordinary composite picture are:

- 1. The streamlines do not follow the body shape perfectly, but always gently cross and recross the fish's form. In presently known flow over aerodynamic shapes, recrossing cannot take place.
- 2. The angles of attack upon the body and fins clearly create propulsive lift.
- 3. Relative velocities of the streamlines "caught" between the vortices and the fish's body are lower than those of the othersbeing slowed by the rotation of the fluid in the vortex. This creates a higher static pressure than ambient, in conformance with Bernouli's law. Thus the high-pressure regions from this analysis occur right where they should be if the vortices are exerting centrifugal forces on the fish.
- 4. The relative flow lines, after leaving the tail, are nearly perpendicular to the path of the fish's motion. This accounts for the nearly perpendicular motion of some of the particles in Run III.
- 5. This picture of streamlines, unlike that over a rigid body, does not stay constant with time. The next instant will show a different set of lines, and a different configuration of forces, but with the same basic phenomena.

The differences between the currently accepted concept of smooth laminar flow over most of the fish's body as exemplified by Fig. 2 and the flow picture Fig. 34 derived from experimental evidence is quite remarkable.

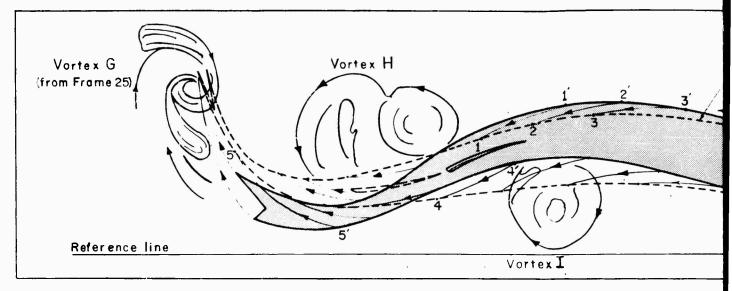


FIG. 34. Relative Flow Vectors, Plotted Between Identical Points accurately oriented tracing of Frames 23, 24, and 25 of Run I (Fi

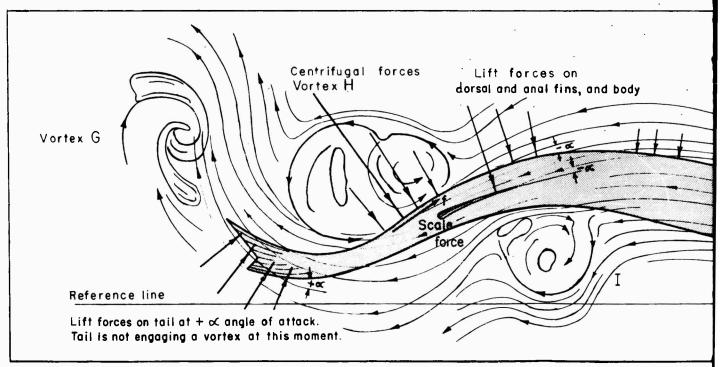
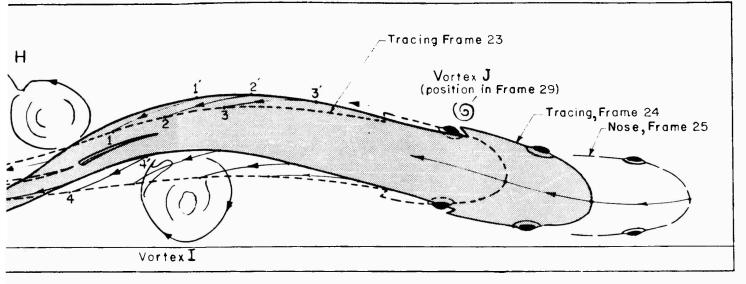
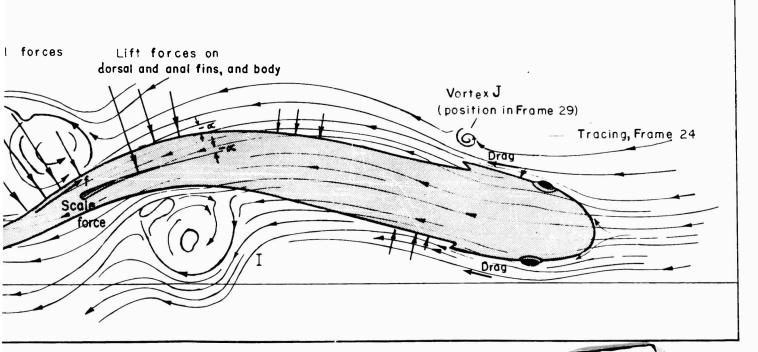


FIG. 35. Relative Streamlines and Forces at Moment of Frame Derived From Fig. 34.





e Flow Vectors, Plotted Between Identical Points on Fish. Combined, ted tracing of Frames 23, 24, and 25 of Run I (Fig. 6).



tive Streamlines and Forces at Moment of Frame 24, Run I (Fig. 6) Fig. 34.

#### CYLINDRICAL AND HOURGLASS VORTICES

The internal structure of the vortex peg has been assumed to resemble a barrel. There are perhaps other possibilities which are depicted in Fig. 36 and 37. One of these is the supposition that the shape might be like a cylinder with the sources feeding in at the center line of the fish and circulating toward the ends. Another configuration might be spirals whose two halves are shaped like an hourglass; the feed might be at the center, spiralling out to the upper and lower ends. Less likely (for hourglass shape) is a feed at the ends, spiralling in contracting loops to the center.

#### EVOLUTION OF A VORTEX

Most mathematical studies of vortices assume a stable condition with respect to time. This is not the case for the fish's vortex pegs.

Each vortex is born, grows and develops, and gradually disappears. Thus it undergoes an evolution with respect to time. The sequence of events, as conceived, is as follows:

Birth

An induced vortex is born, generated by the fish's head-twisting and resulting cross-flow. The fish expends a small initial energy to produce it.

Development

The sides of the fish drag more fluid into the vortex, spinning it in the same direction as when born, and adding more kinetic rotational energy to it. The wave form of the fish now presents an inclined surface to the vortex, preventing it from being dragged forward as an entity. The pressure at the center of the periphery rises well above ambient.

Expansion

The vortex now expands, exerting substantial centrifugal pressure forces against the dorsal

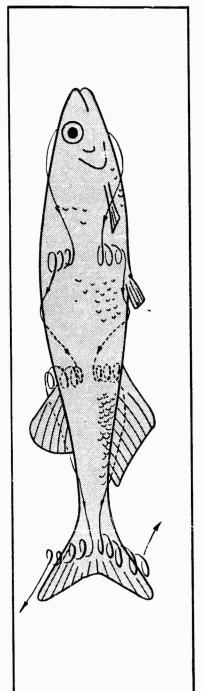


FIG. 36. Possible Cylindrical Vortex Structure, Spiraling Away From Fish Center.

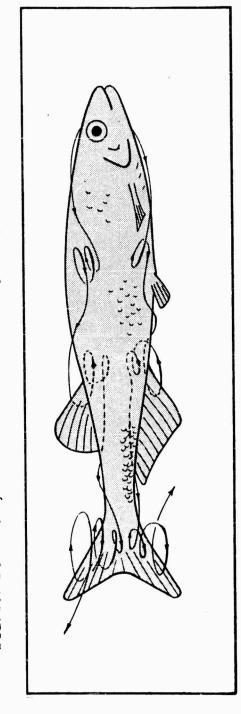


FIG. 37. Possible Hourglass Vortex Structure, Spiraling Away From Fish Center.

fin, anal fin, and the body. It now gives up to these surfaces, which are gliding past it obliquely, a portion of its energy. The wave form of the fish moves backward relative to the vortex center, while the surface of the wave glides forward in its curved path. The particles in the core are rotating rapidly.

# Final Expansion

The vortex, still giving up energy, expands against the tail which envelops and swipes it, not too far from its core. The angle which the tail presents is such that pressures in the vortex create a tail force well-directed to the front. By Newton's laws of action and reaction, the vortex now should be forced to move rearward as it reacts against the tail. However, instead of moving its center, it unwinds an arm obliquely to the rear, thus providing itself the required mass reaction force. When the tail leaves the vortex, the swirl should then theoretically jump forward under the influence of reaction from the arm. However at this moment it unwinds its second arm, throwing it in the forward direction opposite to the first arm and following the tail, thus providing a last parting push and then the mass reaction to keep itself from moving.

## Quieting

We now have the beautiful twin-armed spiral galaxy which rotates very slowly, linking its arm with its predecessor to form the wavy trail. Its pressure has fallen almost to ambient, and it has given up the larger part of the kinetic and pressure energy it once possessed. The velocity of its particles is quite low, and the water in the main arms drifts slowly in alternate directions, largely perpendicular to the fish's path. The energy in these quiet slow orderly spirals represents the energy the fish has expended to propel itself.

This sequence of events has been diagrammed in Fig. 38 in stages a, b, c, and d. The motion of the fish is to the right, and the upper diagrams indicate the variation of static pressure within the vortex.

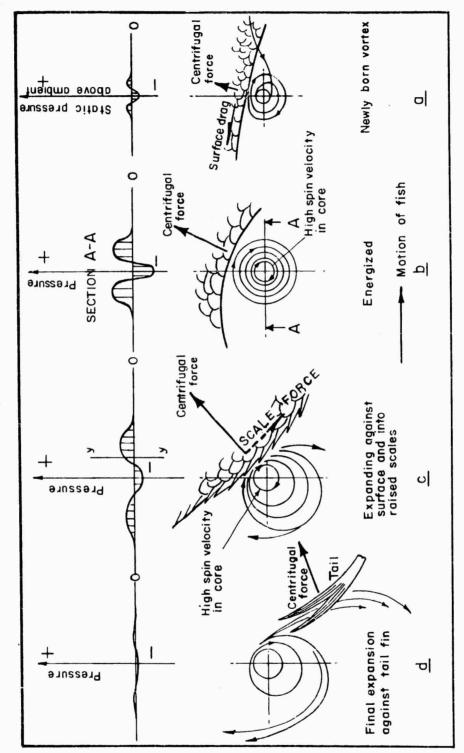


FIG. 38. Pressures and Forces of a Fish Vortex as a Function of Age.

#### WHY DON'T THE VORTICES MOVE?

We see from the above that the vortex balances its reaction on the fish by throwing out a spiral arm mass to the rear. And when the tail has passed, another spiral arm is thrown forward to balance the first. The vortex then does not move except for rotation of its arms.

The writer has observed fish in water with suspended particles of dirt. A steadily swimming fish does not seem to create much disturbance. But when a fish executes a sharp turn, or suddenly accelerates from a stop, a strong current moves in the reverse direction. This current is the arm of a vortex.

A good example of such a current is the arm of Vortex H. Observe this vortex in Fig. 6, Frames 21 through 27. After generation it sweeps obliquely aft in a large, swiftly growing dark spot. Close inspection shows that the large dark spot is not the center, but the after arm, suddenly grown oversize and thrown out. Why is this action different from the others? We shall soon see.

Measuring nose positions accurately, we find the following:

In Frame 22, nose speed = 16.9 in/sec In Frame 23, nose speed = 20.6 in/sec

The fish has therefore accelerated between these two frames at the rate of:

Acceleration a = 
$$\frac{20.6 - 16.9}{0.01 \text{ sec}}$$
 = 370 in/sec<sup>2</sup>

$$a = 31 \text{ ft/sec}^2$$

In Frame 24 the fish reaches 22.0 in/sec. It is believed that it is capable of considerably greater accelerations than this. Nevertheless, we see that the rearward spurt of the Vortex H arm coincided with an acceleration of the fish. Thus the writer presents two observable facts:

- 1. A rearward current is created when the fish accelerates or turns around.
- 2. There is no net rearward current when the fish is swimming at constant velocity.

The last proposition, fantastic though it seems, does not violate Newton's law of action and reaction. If we observe the wake of a ship we will find that there is a strong current of water (the slip stream) leaving the propeller with a high rearward velocity, creating the thrust. However there is another part to the wake: the boundary-layer water which the hull has dragged with it to create a forward-moving current. The total momentum change of the forward current exactly equals the momentum change of the rearward slip stream. This can be expressed mathematically:

$$\int dM_1 \cdot \Delta V_1 = \int dM_2 \cdot \Delta V_2$$

where

 $dM_1$  = element of mass flowing rearward from propeller

∆V<sub>1</sub> = velocity gain of propeller slip-stream mass element

dM<sub>2</sub> = element of mass in forward-moving boundary layer

 $\Delta V_2$  = velocity gain of boundary-layer mass element

After the ship passes, imagine that mass  $M_1$  moving aft engages mass  $M_2$  moving forward. Since their momentums are equal but opposite, their combined linear momentum is zero. Thus if the two mixed perfectly, it would leave them with no rearward or forward motion. In the case of a ship, the mixing is turbulent and disorderly.

In the flow about the fish, there is no well-defined propeller slip stream, nor is there a well-defined boundary-layer current moving forward. Instead, the production of thrust and drag are so intimately interwoven that momentum exchange occurs largely on the same particles, rather than there being two different streams. Thus the laws of momentum can be satisfied, and at constant fish speed no streams need move in the after or forward directions.

## HYPOTHESIS OF PROPULSIVE FORCE FROM SCALES OF FISH

The functions of the scales of a fish are understood by biologists. No doubt it has been wondered whether the scales, in addition to their biological purposes, might not also play a part in propulsion. Of course dolphins, whales, and seals have no scales.

A hypothesis of scale action is presented here, for which the writer has no physical evidence but which nevertheless is intriguing.

The scales lie on the fish in an overlapping manner similar to shingles. It is easy to visualize that on the inside of a flexural curve, the "shingles" lie tight and flat. When the scales are on the outside of a flexural curve, it is possible that they open out to some degree. Perhaps their opening is not enough to be seen, but enough to make the surface rough when it is rubbed in the forward direction.

The photographs show that the vortices, at the time that they are exerting pressures on the fish's body, are very close to the convex surfaces of the flexural waves. As the vortex begins expansion, the evidence shows that the surfaces glide closer and closer to the core until finally the tail fin seems to pass quite close to center just before disengaging.

It is known that a forced vortex, if it ceases to receive energy, will gradually change itself to a free vortex in which, as mentioned before:

Vr = a constant

(particle velocity) × (distance to axis) = constant

We know, therefore, that as we go toward the core of an expanding vortex the particle speed should increase. It is not beyond

possibility that at a certain radius in the energized vortex the particle velocities may be higher than the speed of the scales gliding past; and also that the static pressure at this point may still be above ambient. We must remember that this vortex has a higher energy than ambient.

If the water-spin velocity is higher than the glide velocity of the scales, then the scales will open up due to the curvature of the body and the momentum of the water entering between them. The opened scales will then present a series of turbine-like blades against which the high-velocity water may flow and exert a forward thrust.

The evolution diagram (c of Fig. 38) may assist in clarifying this thought. Line y-y in the pressure diagram above sketch c represents the fish's surface where pressure is above ambient but spin velocity high.

Figure 39 is an enlargement of the fish's actual form and vortex in Frame 13, Run I (Fig. 5). The scales are shown slightly raised under the vortex, although there is no proof of such raising. However, note how close to the core these scales are passing. If they are raised even slightly, turbine action will take place and a scale force f will be added to the centrifugal forces of the vortex. In this way, even more energy can be re-extracted from the vortex. Note that the resultant force R leans more decidedly in the forward direction than the centrifugal force alone.

The sensory nerves in the side of the fish enable it to place its surfaces at the location in the vortex where it will feel its scales lifting. If it approaches too close to the center, the low pressures there will create a drag on the surface which it will feel. Thus its organism can feel and automatically adjust its flexural waves to place them in the right position in the vortex, at the right time, to get maximum return of thrust.

Perhaps an experimenter some day, using a large fish with large scales, will be able to observe the momentary opening of the scales under a vortex.

It should be realized that the Vortex Peg Theory does not depend on the existence of the scale force.

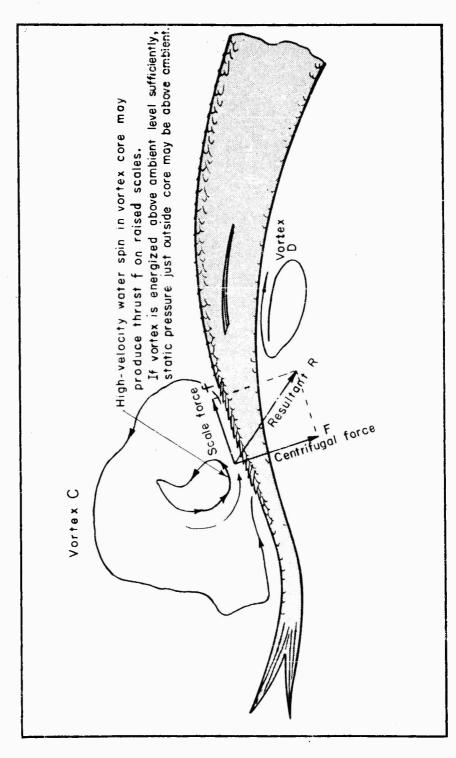


FIG. 39. Scale-Thrust Hypothesis. Body shape and vortices are accurate tracings of enlarged Frame 13, Run I, photograph (Fig. 5).

## CALCULATING DRAG FROM THE VORTEX

Several equations have been developed in fluid-dynamics literature which give the drag required to produce a Karman vortex street. It is interesting to see what might result from their application--even though the fish vortices are not the Karman system. Some of these equations are:

(1) Drag coefficient 
$$C_{D(x)} = 1.6 \frac{u}{V} - 0.64 \frac{u^2}{V^2}$$
 (Hoerner, Ref. 6) where

Drag D = 
$$C_{D(x)} \cdot \frac{\rho V^2}{2}$$
 · xb (Hoerner, Ref. 6)

(2) Drag coefficient 
$$C_D = 1.587 \cdot \frac{u}{V} - 0.628 \frac{u^2}{V^2} \cdot \frac{x}{d}$$
 (Durand, Ref. 7)

where

b = length of body making vortices

d = diameter

u = velocity of vortex group following the body

V = velocity of body making vortices

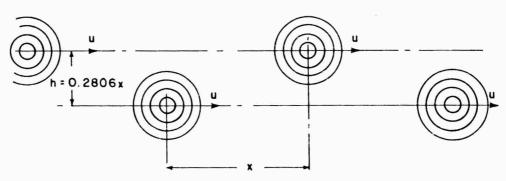


FIG. 40. Karman Vortex Street.

In the case of the single row of vortex pegs which are generated by the fish, the velocity u = 0, since the fish vortices do not move. Consequently, as calculated by either Eq. 1 or 2

## Coefficient of drag $C_D = 0$

This seems to be a reductio-ad-absurdum. But in view of the momentum exchange action discussed in the section "Why Don't the Vortices Move?" it may not be too far removed from fact. There may be no such thing as a total drag on the fish since the processes of drag and thrust are so interwoven that they are almost indistinguishable. The term drag probably can be applied correctly only to the forward one fourth of the fish's body.

## EQUATION OF FISH VELOCITY

"With motion irrotational in fluid incompressible,
A tiny little minnow swims along a line of flow
And the greater its velocity--well, cutting out verbosity-The greater its velocity the faster it will go."

This little ditty, written by L. M. Milne-Thomson, has possibly inspired the following bit of mathematics. Should not there be a relatively simple relationship between the velocity which a fish attains, the frequency of his tail beat, and the amplitude or swing of the tail? Fish seem to be able to vary all three independently at will. It has been observed, for instance, that the speed of a fish is not necessarily proportional to the frequency of his tail beat.

Let us make use of our observations in these experiments, in order to make an approach to the problem:

- 1. The mass of the spiral vortex seems, from observation, to reside mostly in its arms just after fish passage. It is not a solid wheel at that time.
- 2. The mass of the arms depends only on their length 1, thickness h, and span b of the tail (or span between anal and dorsal fins). Thus the mass of one vortex m is

(3) 
$$m = lb \cdot \frac{w}{g} h$$
 (span b  $\approx$  depth of influenced water)

3. The length 1 of each vortex arm is proportional only to the amplitude a of the tail swing. Thus

(4) 
$$m = ab \cdot \frac{w}{g}h$$
 (mass of one vortex)

If there are f complete tail beats per second (f = frequency) and there are two vortices per beat, the number of vortices generated per second is 2f. The total mass M of the vortices generated each second is then

$$M = 2fab \frac{w}{g} h$$

Let us now assume that the <u>same water</u> first creates drag and and then is accelerated by the body to create thrust, separating these mentally.

Let

V = forward velocity of fish = initial relative velocity of water

v = absolute speed of water after being dragged forward
 (averaged)

u = water speed relative to fish, upon ejection

Also if vector similitude is assumed between these velocities, regardless of speed, then

(6) 
$$u = kV$$
 where  $k = a$  constant

(7) 
$$v = cV$$
 where  $c = another constant$ 

Relative to the fish, the water velocity is at first V, then V-v, and lastly u. Writing the mass reaction equation for thrust:

(8) Thrust 
$$F = M \left[ u - (V - v) \right]$$

(9) 
$$F = 2fabh \frac{w}{g} (u + v - V)$$

(10) 
$$F = 2fabh \frac{w}{g} \cdot V(k+c-1)$$

Using the drag equation (even though the fish may not agree)

(11) 
$$\operatorname{Drag} D = C_D \cdot \frac{w}{g} \cdot \frac{V^2}{2} A$$

where

C<sub>D</sub> = drag coefficient based on wetted area

A = wetted area

Equating thrust to drag:

(12) 
$$2 fabh \frac{w}{g} \cdot V(k + c - 1) = C_D \cdot \frac{w}{g} \cdot \frac{V^2}{2} \cdot A$$

(13) 
$$V = \frac{4bh(k + c - 1)}{C_D A} \cdot fa$$

This equation states that the velocity of the fish should be proportional to the product of its tail amplitude and the frequency of the tail beat. All the remaining terms may be grouped together as a constant z for a particular fish, or

$$(14) V = zfa$$

where z = fish constant.

(15) 
$$z = \frac{4bh(k + c - 1)}{C_D A}$$

To review the above symbols:

a = amplitude of tail beat (extreme position to center line)

A = total wetted area

b = span of tail (or span between dorsal and anal fin tips)

c = ratio of velocities = v/V

CD = drag coefficient based on total wetted area

f = frequency of tail beat

h = initial thickness of vortex arms

k = ratio of velocities = u/V

z = the fish constant (constant for a given individual animal)

Thickness h will not be exactly invariant when V increases; however, the change is slow.

### TEST OF EQUATION

Since the motion of our fish was recorded with reasonable accuracy, the following data were measured and computed to see whether or not Eq. 14 "works." The most difficult measurement was that of amplitude since the fish's motion is not exactly uniform.

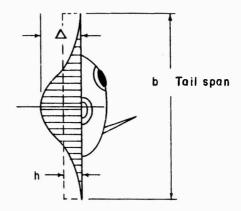
Run No.	Measured Velocity V, in/sec	Measured Frequency f, beats/sec	Average Amplitude a, in.	$z = \frac{V}{af}$ Fish Constant
I, Frames 15 to 31	18. 75	18.8	0.365	2.73
I, Frames 9 to 15	23.2	21.9	0.39 (Frames 9-13)	2.71
II, Frames 22 to 50	10.0	11.5	0.32	2.72
II, Frames 50 to 63	7. 77	8.33	0.33	2.80
III, Frames 24 to 30	17.7	16.7	0.40	2.65

From the above data, it is seen that z seems to be reasonably constant. Although more good data are needed for

complete proof, it is believed that Eq. 13, 14, and 15 are essentially correct.

## FURTHER DEVELOPMENT OF EQUATION

It is now possible to develop the fish equation a step further. Let us make the assumption that h, the average thickness of the vortex arms, is proportional to the thickness  $\Delta$  of the boundary layer. The term "boundary layer" is not correctly applied here but we will use it for want of a better name. Let us assume that  $\Delta$  is greatest at the center line of the fish, and tapers to zero at the tips of the fins in approximately the shape of a bell. This bell shape might approximate the cross section of one vortex arm, equivalent to the area bh in Eq. 3 and 15.



From the above sketch, let us make the crude assumption that

$$h_{\text{(average)}} = 0.44 \triangle$$

The thickness  $\Delta$  and the drag coefficient  $C_{\hbox{\scriptsize $D$}}$  for a flat plate are given in aerodynamics literature. Both are functions of the Reynolds number R .

$$R = \frac{\text{Velocity} \times \text{length}}{\text{Kinematic viscosity}} = \frac{\text{VL}}{\text{V}}$$

Hunsaker and Rightmire (Ref. 8) give the following equations for  $\Delta$  and  $C_{\mbox{\scriptsize D}}$  on a flat plate:

For turbulent flow

(16) 
$$\Delta = \frac{0.376L}{\left(\frac{VL}{V}\right)^{1/5}}$$

(17) 
$$CD = \frac{0.073}{\left(\frac{VL}{V}\right)^{1/5}}$$

For laminar flow, according to the pioneer aerodynamicist Blasius

(18) 
$$\Delta = \frac{5.2L}{\left(\frac{VL}{V}\right)^{1/2}}$$

(19) 
$$C_{D} = \frac{1.33}{\left(\frac{VL}{V}\right)^{1/2}}$$

We may now further examine the fish constant z by means of these expressions for  $h, \Delta$ , and  $C_D$ .

(15) 
$$z = \frac{4bh(k + c - 1)}{C_D A}$$

$$z = \frac{4b(0.44\Delta)(k + c - 1)}{C_D A}$$
(substituting h = 0.44\D)

Substituting again for  $\Delta$  and  $C_D$  in turbulent flow from Eq. 16 and 17

$$\mathbf{z} = \frac{4b(0.44) \left[\frac{0.376L}{\left(\frac{VL}{V}\right)^{1/5}}\right] (k + c - 1)}{\left[\frac{0.073}{\left(\frac{VL}{V}\right)^{1/5}}\right] A}$$

This derivation now cancels out two unknowns, h and C<sub>D</sub>. This is quite desirable since these two quantities are variable with the Reynolds number.

(20) 
$$z = 9.1 \frac{bL}{A} (k + c - 1)$$
 (turbulent case)

Using the same process, but with Eq. 18 and 19 for laminar flow, the following results:

(21) 
$$z = 6.9 \frac{bL}{m} (k + c - 1)$$
 (laminar case)

A noteworthy circumstance is now confronting us. In Eq. 20 and 21 we find that z for turbulent flow is essentially the same as z for laminar flow, the difference lying only in the numbers 9.1 and 6.9. We can again thus surmise that laminar flow or turbulent flow do not greatly alter the fish's method of propulsion. These numbers may possibly represent another constant that is the same for all fish. Or they may possibly represent the upper and lower limits of such a constant depending on the species of animal.

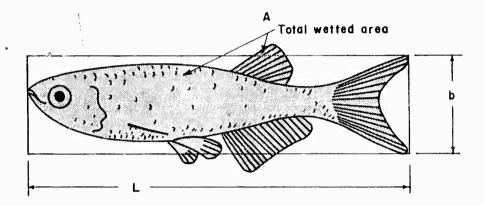
Let us signify this number by )-(.

+ ( = 9 to 7 nominally, but may be of a widely different
value depending on the actual cross section of the
vortex arms

An interesting ratio appears in Eq. 20 and 21. This is bL/A where b = tail span, L = length, A = total wetted area. If a rectangle circumscribes the fish and its tail, it will have the area bL.

Thus the expression bL/A is the ratio of this rectangular area to the wetted area of the animal. Let us dub this area ratio  $\Psi$ .

(23) CoeFishent 
$$\Psi = \frac{bL}{A}$$



The CoeFishent may also have various values for different species of swimming animals; or it may be approximately the same for a variety of species. The writer has few data on this at present.

We may now write our final equation for z. Using the symbols of Eq. 22 and 23 and substituting into Eq. 20 and 21 we have another form for the fish constant, added to our previous equations:

(24) 
$$z = \frac{1}{2} (x + c - 1)$$

or

(15) 
$$z = \frac{4bh(k + c - 1)}{C_D A}$$

where

(14) 
$$Velocity V = zfa$$

- z = fish constant = approximately 2.65 to 2.8
- +( = Pisces number (value unknown, perhaps in the order of 9 to 7)
- ₩ = CoeFishent (tail span · length/total wetted area)
- f = frequency of tail beat
- a = amplitude of tail beat

The writer suspects that the variation of +,  $\forall$ , k, and c between different species of fish may be such that z, the fish constant, may well be a fixed number for many species.

#### CONCLUSIONS

The water flow about a swimming fish has been found to be an evolutionary system of vortices. These are first generated at the sides near the fish's gills. They are large, well-formed, discrete entities, spinning about vertical axes. In flowing along-side they grow in size, one vortex contained in each moving concave flexural wave of the fish's body.

Before the fish has passed, the vortex centers align themselves in a single row to the path of the fish's head. After passage there is no appreciable movement of these centers in the forward or aft direction. Nor is there a rearward moving current in the fish's wake when it is swimming at steady speed. This is unlike the wake of man-made vessels, where a strong rearward current flows from the propellers.

It is observed that the water motion about the ever-changing form of this swimming fish does not correspond at all with the concept that is presently accepted--of laminar parallel flow over most of the body.

A cross - re-cross flow phenomenon was discovered, the water changing from one side of the fish to the other, and possibly back again. Such flow occurs in between the vortices. This in itself would render present methods of drag computation incorrect for the undulating form of a fish or a dolphin.

The present methods, however, are valid for determining the resistance of a fish in a glide, i.e., when it is not undulating. This has been verified in the case of mackerel and herring by E. G. Richardson (Ref. 9) who found that the resistance of these fish when dead was close to that of wooden models of their bodies.

The theories of the Vortex Peg Hypothesis and the Scale Force Hypothesis postulated by the writer state that propulsive forces are exerted by the vortices on approximately two thirds of the fish's body as well as on its after fins and tail surfaces. These forces have their origin in the centrifugal force fields existing within each vortex, and in the high angular momentum of the fluid near the core of the vortex.

Such forces do not exist about rigid streamlined shapes to any great degree. Thus, presently known methods for calculating the drag of rigid streamlined shapes do not seem valid for calculating the resistance of an undulating fish, whether turbulent drag coefficients or laminar drag coefficients are used.

The astonishing elliptical and two-armed vortex generated by the leaping dolphin apparently leads to one conclusion. It is probable that these cetaceans (dolphins, porpoises, and whales) generate and propel themselves by the same type of vortex as does the tiny fish--according to the Vortex Peg Hypothesis. This is irrespective of the large difference in Reynolds number. The only difference is that in the cetaceans the vortices will form above and below the body and spin about horizontal axes since their tails lie in a horizontal plane and their bodies flex up and down.

The equation of fish velocity, as proposed, states that the velocity V of a fish is given by

V = fish constant X frequency of tail beat X amplitude of tail

The equation corresponds well with the measurements made in these experiments. Further verification on other fish should be made.

From the standpoint of locomotion, the Vortex Peg and the Scale Force Hypotheses postulate the fish as utilizing a means of propulsive action unlike that which has heretofore been imagined possible in a fluid medium. It is a propulsion method that first generates in the water near the gills, a flow that may be thought of as a machinery system of large fluid flywheels. These wheels roll in contact with the fish's body and fins. The energy put into these flywheels to create them and set them in rotation is assumed to be largely regained by the fish, due to its ability to synchronize its flexible body with them. This synchronization of its undulations is such that it takes advantage of the centrifugal forces existing within each wheel. The high velocity of the water inherent in the central regions of a natural vortex is believed to exert a turbine bladelike action on the scales or skin of the fish. Both forces drive the fish forward.

The presently known concepts of fluid dynamics--turbulent flow and laminar flow--apparently seem incapable of logical explanation for the speed of fish and dolphins. Are we justified in surmising that the phenomena observed in these experiments shows a third type of fluid motion? If, as is postulated by the two hypotheses, otherwise lost energy is being made available to recirculate between fish and fluid, an appropriate name for this third fluid process might be Regenerative Vortex Flow.

Thus, it is not a question of turbulent flow or laminar flow about the body of the animal. It is rather that Nature's creatures apparently are able to marshal to their aid propulsive forces from a hitherto unsuspected kind of fluid dynamic phenomenon, one which for its efficiency depends upon the properties of a natural vortex, and on cooperation between the living organism and the fluid.

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